

Math 321 – Spring 2019 – Some review for quiz 1 from chapters 1 and 2

Let there be a collection of tokens with four colors {red, blue, green, black, yellow} and each color having dollar values {\$1, \$5, \$10, \$20, \$50, \$100}. Calculate the following.

1. Draw 3 tokens without replacement. $P(\text{all same \$ value})$
2. Draw 3 tokens without replacement. $P(\text{all same color})$
3. Draw 3 tokens without replacement. $P(\text{exactly two are same \$ value})$
4. Draw 3 tokens without replacement. $P(\text{exactly two are same color})$

Let there be a collection of tokens with four colors {red, blue, green, black, yellow} and each color having dollar values {\$1, \$5, \$10, \$20, \$50, \$100}. Calculate the following.

1. Draw 3 tokens without replacement. $P(\text{all same \$ value})$

Solution: First we need to count the total number of possible outcomes. Since we just want the tokens to be the same value, the order they are drawn in does not matter. So we are working with combinations. There are 6 \$ values and 5 colors of each dollar value, thus there are 30 total tokens. There are $\binom{30}{3}$ total possible outcomes.

Here is how we will decide how many ways we can get all 3 tokens the same \$ value:

- First, choose the \$ value that will be shown on all 3 tokens (there are 6 \$ values to choose from). Thus there are $\binom{6}{1}$ ways to accomplish this.
- Second, choose 3 tokens with that \$ value (there are 5 of each \$ value since there are 5 colors). Thus there are $\binom{5}{3}$ ways to accomplish this.

We then use the multiplication rule to get the total number of ways that all 3 tokens can be the same \$ value and divide by the total number of ways to draw a subset of 3 tokens.

$$P(\text{all same \$ value}) = \frac{\binom{6}{1} \binom{5}{3}}{\binom{30}{3}} \approx 1.48\%$$

In R:

```
> choose(6,1)*choose(5,3)/choose(30,3)
[1] 0.01477833
```

2. Draw 3 tokens without replacement. $P(\text{all same color})$

Solution: Here is how we will decide how many ways we can get all 3 tokens the same color:

- First, choose the color. $\binom{5}{1}$
- Second, choose 3 tokens with that color. $\binom{6}{3}$

$$P(\text{all same color}) = \frac{\binom{5}{1} \binom{6}{3}}{\binom{30}{3}} \approx 2.46\%$$

In R:

```
> choose(5,1)*choose(6,3)/choose(30,3)
[1] 0.02463054
```

3. Draw 3 tokens without replacement. $P(\text{exactly two are same \$ value})$

Solution:

- First, choose the \$ value. $\binom{6}{1}$
- Second, choose 2 tokens with that value. $\binom{5}{2}$
- Third, choose the last token of a different value, any color. There are 30 total tokens, the value that we drew 2 of has 5 colors, so we exclude those 5 tokens of that value to get 25 total left to choose the last token from. $\binom{25}{1}$

$$P(\text{exactly two are same \$ value}) = \frac{\binom{6}{1} \binom{5}{2} \binom{25}{1}}{\binom{30}{3}} \approx 36.95\%$$

In R:

```
> choose(6,1)*choose(5,2)*choose(25,1)/choose(30,3)
[1] 0.3694581
```

4. Draw 3 tokens without replacement. $P(\text{exactly two are same color})$

Solution:

- First, choose the $\binom{5}{1}$
- Second, choose 2 tokens with that color. $\binom{6}{2}$
- Third, choose the last token of a different color, any value. $\binom{24}{1}$

$$P(\text{exactly two are same \$ value}) = \frac{\binom{5}{1} \binom{6}{2} \binom{24}{1}}{\binom{30}{3}} \approx 44.33\%$$

In R:

```
> choose(5,1)*choose(6,2)*choose(24,1)/choose(30,3)
[1] 0.4433498
```