Exam 2 review (draft: 2019/04/07-22:21:51)

Exam 2 covers all material since the first exam. You will still need to remember much of the material since the first day of class though, including, but not limited to, many of the rules of probability.

My recommendation is to study the two quizzes, make sure you understand everything on those perfectly. Make sure you have looked over any example problems included in the course notes that I have posted online (for chapters 3 and 4). Make sure you understand the webwork homework problems. (also note that the chapter numbers for the webwork assignments do not necessarily align with the chapter numbers for my notes). Of course, you should also understand everything and all examples covered in class. Sometimes these vary somewhat form what is seen on homework or in my online notes. You should also pay attention to what you did in the R Activity assignments.

Here is a summary list of topics (note that this list is not completely exhaustive):

- Random variables (discrete and continuous)
- Probability distributions (pmf, pdf, cdf)
- Expected value and variance of RVs
- Binomial, geometric, Poisson, exponential, normal
- Central limit theorem and law of large numbers

Example/practice problems:

1. Flip a biased coin with probability of heads being 0.2. Refer to getting heads as a success.
   (a) If we flip the coin 20 times, what is the probability of getting 13 heads?
   (b) Now, let's flip the coin until the first head occurs. Write the probability mass function for $X = \{\text{the total number of flips up to and including the first head}\}$. What is the probability that there are 5 tails before the first head?
2. Consider a population of animals where adult weight is normally distributed with mean 7.3 kg and standard deviation 1.8 kg.
   (a) What is the probability that a randomly selected individual weighs less than 5 kg?
   (b) What is the probability that a randomly selected individual's weight will be between 8.2 and 9.5 kg?
   (c) What is the probability that a randomly selected individual's weight is between 5.5 and 9.1 kg?
   (d) What is the probability that a randomly selected individual's weight is above 10.6 kg?
   (e) If a sample of 12 individuals is taken, what is the probability the mean weight will be less than 5 kg?
   (f) What is the probability that the mean weight is exactly 5 kg?
   (g) If an iid sample of 25 individuals is selected, what is the probability that every individual's weight is in the range of 5 to 10 kg?
   (h) What is the probability that $\frac{X_3-9}{1.8} < -1$?

3. Consider a manufacturer producing a thin laminate material for protective coatings of surfaces. The laminate material is produced continually with a width of 1 m and rolled onto spools that hold a total length of 200 m. Assume it is known that on average there are 3 imperfections per 40 m^2.

   (a) If a particular customer is to buy a complete spool of this material, what is the probability that the total number of imperfections is greater than 25?

   (b) If that customer needs at least four 1 m by 10 m sections with no imperfections (a total of 1 m by 40 m), what is the probability that the first 40 m on the spool have no imperfections?

   (c) Imagine that the production machinery is turned on and begins churning out a sheet of laminate and that it will be stopped once the first imperfection occurs. What is the probability that the machine will produce at least 20 m of laminate before being stopped?

4. Consider the pdf $f(x) = a(16 - x^2)$ for $0 \leq x \leq 4$.

   (a) Find a.

   (b) Find the cdf.

   (c) Find $E(X)$.

5. Consider the pdf $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. Note that this is the exponential pdf.

   (a) Find the median.

   (b) Find the first quartile.

6. Consider the pmf

x	0	10	20
$f(x)$	0.10	0.60	0.30

   (a) Calculate $E(X)$ and $Var(X)$.

   (b) Calculate $E(100 + 2X)$ and $Var(100 + 2X)$.

   (c) Write the formula for and sketch a graph of the cdf.

   (d) If two independent X values are sampled (a sample of size 2) are selected, with replacement, what is the probability that the sample mean is 0? $P(\overline{X}_2 = 0)$.

   (e) For the sample of size 2, what is the probability the sample variance is zero?

   (f) What is the probability that the sample mean for a sample of size two that is 10 or less?

7. Consider the uniform random variable $X \sim U(0, 100)$.

   (a) Write the pdf.

   (b) Write the cdf.

   (c) Find $E(X)$.

   (d) Find $P(50 < X < 70)$.

   (e) Find b so that $P(30 < X < b) = 0.6$.

   (f) Find the first quartile.

8. Explain the following R code.

```
> x=c(0,2,4,6,8,10)
  p=c(0.69,0.01,0.05,0.12,0.03,0.10)
  m=sum(x*p)
  v=sum(x^2*p)-m^2
```

9. Explain the following R code.

```
> x=rnorm(100,mean=25,sd=5)
  hist(x,breaks=seq(from=0,to=50,by=2))
```

10. Consider the experiment where we are tracking the time between arrivals of data packets at a server. It is known that the mean time between data packet arrivals is 479 μs (microseconds). A data scientist records the arrival times for data packets, and has collected a list of times for 1 million packets.

    (a) Calculate the approximate probability that the average time between packets is greater than 479.5 μs.

    (b) Calculate the approximate probability that the total time taken to collect this dataset is less than 8 minutes.

    (c) If the scientist continues to collect data on the arrival times of packets, what will happen to the average between packet time in the sample?

    (d) For $n = 10^6, 10^7, 10^8, 10^9, 10^{10}, 10^{11}, 10^9$ calculate the probability that the sample mean is less than 478.95 μs.