

1 Introduction: Populations, Samples, Data, and Variables

Statistics. What is *statistics*? The science of statistics has two primary components: (1) to analyze data (observations of physical populations, processes, or experiments) and to understand the properties of data, and (2) to use data and our analysis of it to understand the underlying population or process that generated the data. This involves understanding variability, uncertainty, and randomness. These are loaded terms, but you probably have an intuitive feel for what they mean. We'll discuss them further later.

Populations. The ultimate goal of statistics is to understand the properties of a *population*. A population is a collection of individuals and can be *concrete* or *hypothetical*. A concrete population is one where all the individuals currently exist. A hypothetical population means that some individuals do not yet exist. In some cases, a population can better be thought of as a process that generates individuals as in the case of a particular experimental or man-made apparatus or a natural process. Table 1 shows examples of concrete and hypothetical populations.

For example, we may be concerned with industrial machinery that produces something. Assuming the machinery has a constant condition and is maintained constantly over time, we might ask, what is this machinery capable of producing? We may have a collection of produced items already, but we are interested in all possible items produced, which is hypothetically an infinite population. We are really interested in the property of that machinery, say it's likelihood of producing items that fail a quality control check, but nevertheless we investigate this by looking at the items it actually produces and how often they fail quality control.

Concrete	Hypothetical
All US currently living citizens	All babies born in the US in the current year (some in the past and some in the future)
items produced by a factory so far this year	all items capable of being produced by a factory (Not all yet exist. Some will never exist! But they are all hypothetically capable of being produced.)
a jar of coins	an infinite sequence of coin flips from a single coin
last year's harvest of a particular crop	all possible harvests of a particular crop
all voters' opinions on a particular political issue	all votes on the next presidential election

Table 1: Examples of concrete and hypothetical populations.

Samples. Instead of studying an entire population, which is often unfeasible due to the size of the population or the cost of such a study, we are usually limited to studying a subset of a population. A *sample* is a subset of a population. In most cases, the goal or hope is that the sample resembles

the population in the trait or quantity of interest. Statistical theory also studies ways to select samples and how to evaluate whether they are good methods.

Data. What is *data*? Data is a collection of observations of some phenomenon. There are two primary types of data: *quantitative* and *categorical*. Quantitative data often result from some kind of measurement, e.g. time elapsed for a given event or the physical trait of an object (length, area, volume, mass, etc.). For data to be quantitative, it must represent a quantity or amount of something. Categorical data is often qualitative in nature and occurs when observations can be organized into a finite number of discrete classes, e.g. color, type, etc. We will primarily be concerned with quantitative data in this course.

Quantitative data can further be described as *discrete* or *continuous*. If all possible values can be counted or at least theoretically enumerated for counting, then the data is discrete. Discrete data might have an infinite set of possible values, e.g. $\{1, 2, 3, 4, \dots\}$, all natural numbers without any obvious maximum cut-off. If data can take any value in some interval, e.g. all real numbers in $[0, 1]$, it is continuous.

Do not get bogged down in worrying too much about classifying data as categorical/quantitative or discrete/continuous. What is important to know is whether a given statistical method is valid for a particular data type. In other words, methods that we will use on continuous data do not always work well on discrete data. This is something that we will discuss for each statistical technique we encounter, and it will generally only become an issue if you go on to take advanced statistics courses.

Variables. Quantitative data is composed of numerical values for a particular variable, we'll usually use the letter X . The variable X might represent something like water well depth (an example we'll look at shortly). Each water well has a particular depth, and different wells have different depths in most cases. Theoretically, a well can take on any depth from, say zero up to some maximum limiting value but such a maximum value is difficult to determine precisely. So we will just say that X takes on values in the continuous interval $[0, \infty)$.

We will refer to X as a *random variable*. This is a concept that we will elaborate on more later in the course, but for now just think of it as being a variable that takes on values over a population, e.g. depth for the population of water wells. In order to find the value a random variable takes, we will need to select an individual in a population and then gather the necessary information, e.g. measure water well depth.

Variable notation. Normally a capital letter X is used for the (random) variable as an undetermined value (i.e. water well depth in general) and a lower case letter x when we are looking at a particular individual in the population, e.g. the depth of a particular well. In other words, X is the variable representing well depth, and when we look at a particular well, X takes on an actual numerical value x . Lower case x here is your traditional variable in the algebra and calculus sense.

A note about continuous vs discrete. The line between discrete and continuous can sometimes be blurry. Consider money as an example. In many circumstances, money can be counted in terms of a smallest unit (e.g. the penny, \$0.01, for US physical currency coins and bills), but in many circumstances it can be counted in fractions of a cent (e.g. prices in the stock market are sometimes given out to 4 or 5 decimal places). The advent of high-value cryptocurrencies (e.g. Bitcoin) further show this.

Categorical	Quantitative
species of trees on a specific plot of land	the number of trees on that plot of land (discrete)
paint color of a vehicle	volume of paint required to coat a vehicle (continuous)
nationality of a person on a plane	the number of passengers on the plane (discrete)
variety of grain planted in a field	total harvest from that grain field (possibly discrete if number of bushels with a strict rounding rule, continuous if volume or mass)
responses to a multiple choice public survey on a particular political issue	proportion of a population that supports a particular political issue (generally continuous, but could be thought of as discrete if we have a fixed population size)
whether or not a manufactured building material failed a quality control check	force required to cause failure of a building material (continuous)
type of lightbulb (e.g. LED, incandescent, fluorescent)	time until failure of a lightbulb (time-elapsed measurements are generally treated as continuous if measuring with reasonable accuracy)
model year of a car (1995, 2018, etc. Note that the year model does not always represent the actual year that the vehicle was manufactured in)	number of years a car has been in service (discrete, if only rounding to whole years, continuous if measuring time with high decimal precision)

Table 2: Examples of categorical and quantitative data. Quantitative variables are noted as discrete or continuous.

Also consider exchange rates and that, at the present moment I am writing this, 1 Venezuelan Bolivar is worth 0.000004 US Dollars. In principle, money could theoretically be subdivided without limit and is often treated as continuous.

Also, measurement is limited by the accuracy of the measuring device (e.g. 1/16 of an inch resolution on a ruler), so one could similarly think of length measurements as only taking on discrete values. Or we could think of length as being the number of molecules along the edge of an object! However, in most circumstances length and most other physical measurements are thought of as continuous.

Our potential measurement accuracy is usually, at least in principle, well beyond the human perceptual accuracy, and this is another justification for treating certain variables as continuous.

A note about categorical vs quantitative. What if our categories have numerical labels? The basic idea is that if doing arithmetic operations on those numerical labels does not generate numerical labels which are meaningful in the given context, then they are truly categorical and not quantitative. A quantitative variable represents a quantity or amount of something.

Consider the example of paint color. What if we give the colors numerical labels? This is still a categorical variable! If we call blue 1, red 2, and green 3, then these numbers are no longer quantitative in the sense that they are simply categorical labels of a qualitative observation. We do have that “ $1 + 2 = 3$ ”, but “red + blue = green” is not meaningful, furthermore “ $2 + 3 = 5$ ” but it is not clear what color 5 should represent. We could easily have used different numerical labels for the colors without changing the underlying nature of the data.

Consider calendar year as an example, e.g. year in which a historical event occurred. It is still categorical because the adding or subtracting two years generally will have no meaning within the given context. Consider two historical events, one in 900 CE (common era) and another in 800 CE. It is true that $900 + 800 = 1700$, but it is not clear how the fact of a historical event in 1700 CE is related to those in the past. If we are counting the number of siblings a person has, adding 2 siblings with 3 siblings to get 5 siblings has a very clear meaning and is thus truly quantitative.

2 Introduction to Rand a preliminary dataset

Example dataset. Let’s look at a dataset. Table 3 below shows all water well depths in a specific region in Spokane County, Washington (Grid location T24N-R45E-S06). Figure 1 shows the Department of Ecology website and location of the wells on a map.

115	107	114	465	250	220	690	280
520	100	450	200	80	500	120	375
575	700	80	640	640	680	260	100
300	160	440	165	660	120	560	180
110	65	120	86	360	100	300	165
500	600	600	260	300	620	480	440
240	120	420	175	585	240	460	200
200	430	482	290	290	620	660	440

Table 3: Well depths (ft) in Spokane County for grid location T24N-R45E-S06.

Let’s explore this data using R. First we need to import the data into R. There are several ways to go about doing this. I will be giving instructions for Windows. It may vary slightly if you are using another operating system.

Set Your Working Directory. First, we’ll want to set our working directory in R. In RStudio you can do this from the main menu with:

Session → Set Working Directory → Choose Directory..

Otherwise we’ll need to set it manually.

Let’s say your directory is `C:\Users\myname\files\statsRwork`, then you will need to type:

```
> setwd("C:/Users/myname/files/statsRwork")
```

Notice that the slashes have reversed direction!!!

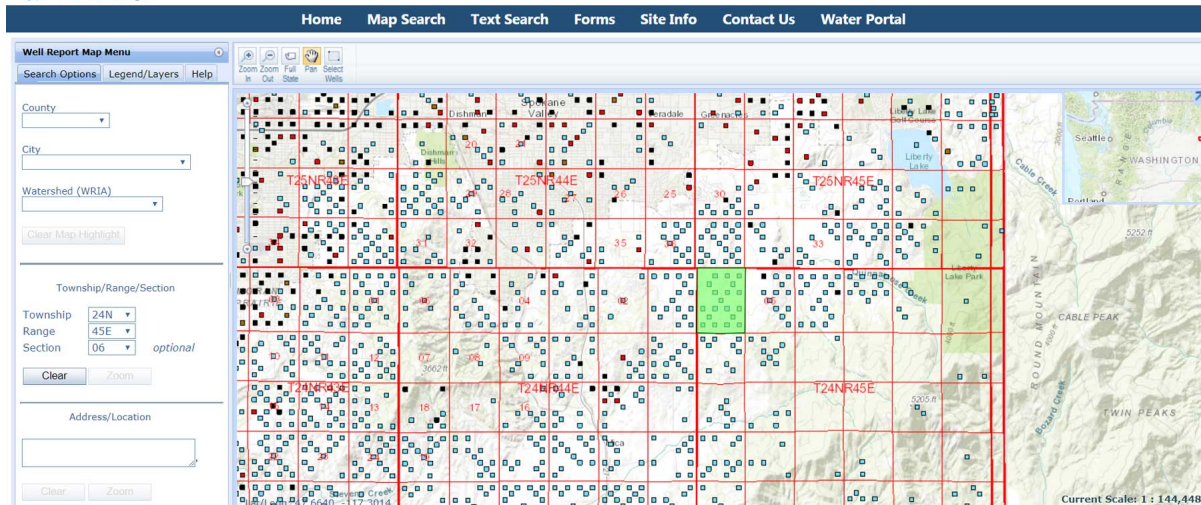


Figure 1: Washington State Department of Ecology website showing the locations of water wells for a portion of Spokane County. Grid location T24N-R45E-S06 is highlighted.

Import a Dataset. To import a single list of data into R, here is the table read method of importing data:

1. Open an Excel spreadsheet.
2. Put a name for your variable/data in the first cell (recommended text only, no spaces or special characters).
3. Type all data values in the column under the name.
4. Save as filetype “Text (MS-Dos) (*.txt)”. Alternatively, open it in MS Notepad, and copy and paste the entire data column, including the name, into Notepad and save it as a *.txt file. Let’s say we saved it as `filename.txt`.
5. Open R and execute the command:


```
> x=read.table("filename.txt", header=T)
```

Alternatively, if the dataset is small enough that manually typing out the entire list is not a major fuss, we can just use the R command

```
> x=c(x1,x2,x3,...,xn)
```

where `x1,x2,x3,...,xn` is our list of actual numerical data values.

I have saved our list of water well depth data as `wellddepth.txt`. Figure 2 shows how your data should look in Excel and Notepad.

So we can execute:

```
> wd=read.table("wellddepth.txt",header=T) This will give our data a “table structure”. To extract just the numerical data, we’ll need to execute
> x=wd$depthdata
```

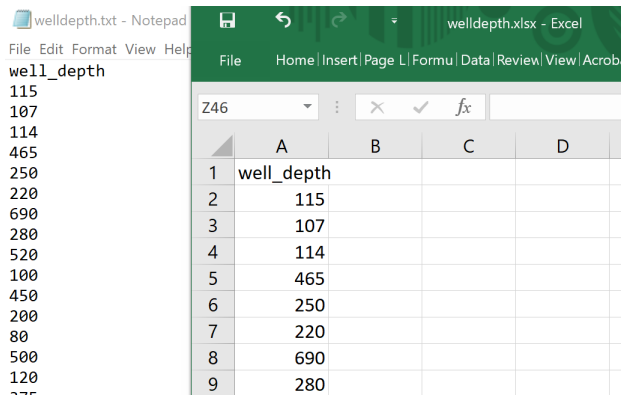


Figure 2: Water well depth data shown in an Excel spreadsheet column with label and in a Notepad text file.

The name “depthdata” is the header name for our column of data. In R, the dollar symbol, \$, is used to reference the actual data under a given name. A table of data could actually have multiple columns with each column given a unique name, e.g. a table named `y` with columns named `col1`, `col2`, `col3`. We can extract the data from the 2nd column by

```
> y$col2.
```

We can also import our well depth data directly by typing it all out separated by commas inside of a `c()` with:

```
> x=c(115,107,114,465,250,220,690,280,
      520,100,450,200,80,500,120,375,
      575,700,80,640,640,680,260,100,
      300,160,440,165,660,120,560,180,
      110,65,120,86,360,100,300,165,
      500,600,600,260,300,620,480,440,
      240,120,420,175,585,240,460,200,
      200,430,482,290,290,620,660,440)
```

Now we have our list of well depths stored as `x`, and we are ready to analyze it with statistical techniques.

3 Descriptive statistical methods

Descriptive Statistics. Now we will calculate some numerical summaries that will give us a feel for the “shape” of this dataset. Any data point or summary piece of information about a population or sample is called a *statistic*.

3.1 Measures of central tendency and location

The idea of *location* and *central tendency* entails that we want to understand where the data lies, e.g. on a numberline or within a categorical list, and what the typical data value is.

You are probably familiar with the concept of an “average” or “mean.” This is often referred to as a measure of central tendency and can be thought of as the central, typical, or expected data value. This is not the only possible way to describe the central tendency or typical data.

Mode. For categorical or discrete data, the *mode* is a useful measure of central tendency. The mode is the most common data point.

Example: Consider the dataset: $\{1, 2, 3, 3, 3, 3, 3, 6, 7, 8, 8, 9\}$. There are five 3’s, and this is the most common data value, thus 3 is the mode. If there were another data value that also occurred five times in the dataset, then the dataset would be called *bimodal* (two modes) or *multimodal* (three or more modes).

Median. For quantitative data, the *median* is a useful measure of central tendency. The median can also be used for categorical data when the data has a natural order to it, e.g. letter grades or a ranked preference scale.

The median will be denoted with a “~” (referred to as a ‘tilde’) over a letter. The population median is denoted $\tilde{\mu}$ and the sample median is denoted \tilde{x} .

Consider dataset $\{x_1, x_2, \dots, x_n\}$. This dataset may not necessarily be in increasing numerical order. We need to sort it, and thus create the *order statistics* $\{x_{(1)}, x_{(2)}, \dots, x_{(n)}\}$ which is just the same dataset, but written from smallest to largest in numerical order, i.e. $x_{(1)} = \min\{x_1, x_2, \dots, x_n\}$, $x_{(i)} \leq x_{(i+1)}$ for $i = 1, 2, \dots, n - 1$, and $x_{(n)} = \max\{x_1, x_2, \dots, x_n\}$.

To sort a dataset in R and create the order statistics use the following command.

```
> sort(x)
```

R tip: If you have a dataset stored as **x**, you can overwrite **x** with its sorted list by doing:

```
> x = sort(x)
```

Generally, there will not be a reason to keep your dataset unordered thus you can replace it by its order statistics in this way.

Once you have your dataset sorted in increasing order, you can calculate the sample median as

$$\tilde{x} = \begin{cases} x_{(\frac{n+1}{2})} & \text{if } n \text{ is odd} \\ \frac{1}{2} \left(x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)} \right) & \text{if } n \text{ is even} \end{cases}$$

This gives the middle value of your sorted dataset if odd in length, or average of two middle values if even in length

```
> median(x)
```

R tip: Check your sample size with the following command.

```
> length(x)
```

R tip: Here is an alternative way to manually calculate the median:

```
> n = length(x)
```

```
> x[n/2] (if n is even)
```

```
> x[(n+1)/2] (if n is odd)
```

Mean. For quantitative data, the *mean* is another measure of central tendency. The mean is denoted by a ‘bar’ over a letter. The population mean is $\bar{\mu}$ and the sample mean is denoted \bar{x} .

The mean coincides with the common concept of ‘average.’ Note that there are multiple types of averages though! We are interested in the *arithmetic average* here.

The sample mean is calculated by adding up all data values and dividing by the sample size:

$$\bar{x} = \sum_{i=1}^n x_i$$

This can be accomplished in R by:

```
> mean(x)
```

We can also calculate the mean by:

```
> sum(x)/length(x)
```

The mean is a statistic that is thought to give us some information about the sample it was calculated from, and we might even believe it gives us some information about the underlying population from which the sample was taken. In this sense, the mode, median, and mean are all “summary descriptive statistics.” They are statistics that describe the central tendency of the data, or the most likely or most common observation.

Quartiles. Quartiles are also a measure of location. They are not generally thought of as describing central tendency. They split the data into chunks of 25% by count, e.g. a dataset of 12 points would be split into groups of 3 by the quartiles.

The median is actually the second quartile. The first quartile is the median of the first half of the data and the third quartile is the median of the last half of data. If n is odd, I prefer to exclude the median and use the first and last $(n - 1)/2$ data points for calculating the quartiles. It is a valid technique to include the median as well. Don’t get too hung up on which method to use, just pick one and stick with it.

```
> q1 = quantile(x,0.25)
```

```
> q3 = quantile(x,0.75)
```

Note that there are 9 different ways to calculate quartiles (or more generally, quantiles) in R, the default is `type=7`. Try `> q1 = quantile(x,0.25,type=2)` for example.

Normally, the quickest way to get the above summary statistics is with the `summary()` command. It gives the minimum, maximum, mean, median, and quartiles.

```
> summary(x)
```

The minimum, 1st quartile, median, 3rd quartile, and maximum are referred to as the *five number summary* and can be calculated also by:

```
> fivenum(x) (similar to summary() but only gives a list of min, q1, median, q3, max)
```

Quantiles. These are a sort of generalization of quartiles. Instead of splitting the dataset up into 4 parts, each containing 25% of the data, we can split it up into any number of sections. Quintiles for 5 sections at 20% each (this is actually quite common in economic data such as household income), deciles for 10 sections, percentiles for 100 sections. The k -quantiles split the data up into k sections, i.e. $k = 4$ for quartiles, $k = 5$ for quintiles, $k = 10$ for deciles, and $k = 100$ for percentiles. To calculate the j^{th} k -quantile in R:

```
> quantile(x,j/k)
```

Trimmed mean. If the median and mean of a dataset are very different, that is an indicator that there may be some outliers or a significant asymmetry in the way your data is distributed over the

set of all possible values. Sometimes trimming off extreme data values on the upper and lower end, and then calculating summary statistics can be useful.

A $\alpha\%$ trimmed mean is calculated by removing the $\alpha\%$ smallest and $\alpha\%$ largest data values then calculating the mean of the resulting reduced-size dataset and is denoted $\bar{x}_{tr(\alpha)}$.

Example: Consider the dataset $\{1, 1, 2, 4, 5, 8, 9, 10, 20, 30\}$ with mean $\bar{x} = 9$ and median $\tilde{x} = 6.5$. Since we have exactly 10 data points, a 10% trimmed mean would remove the largest and smallest point to get $\bar{x}_{tr(10)} = 7.375$, and a 20% trimmed mean would remove the two largest and two smallest values to get $\bar{x}_{tr(20)} = 6.333$. These are closer to the median.

An $\alpha\%$ trimmed mean can be calculated in R as:

```
> mean(x, trim= $\alpha$ /100) (e.g. a 10% trimmed mean is mean(x, trim=0.1))
```

3.2 Measures of variability and spread

Calculating a mean or median does give you some information about a dataset, but in any interesting case, the data will not all be identical and will vary. Some data will fall to one side or the other any of the measures of center or location. It is also important to understand how data varies! There are several ways to characterize how data is spread out and varies over the set of all possible values.

Range. The difference between the maximum and minimum data value is one way to understand the spread of the data and is called the *range*:

```
> min(x)
> max(x)
> range(x) (this give both the minimum and maximum, not their difference)
> datarange = max(x)-min(x) (or alternatively: datarange = range(x)[2]-range(x)[1])
```

Interquartile range. The interquartile range is defined as the difference between the third and first quartiles $IQR = Q_3 - Q_1$ and is also a measure of the spread of the data. It is generally better than the range as a measure of spread since it will not be as affected by extremely large or small data values.

```
> IQR(x)
```

Similarly, as a measure of spread, you can take the difference between various quantiles.

Variance and standard deviation. Next to the mean and median, probably the most common descriptive statistics are the *variance* and *standard deviation*. These are both measures of how the data varies. The population variance is denoted σ^2 and the population standard deviation is the square root of variance σ . Sample variance is denoted s^2 , and sample standard deviation is denoted $s = \sqrt{s^2}$.

Sample variance is calculated by the formula

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

In R this can be achieved by:

```
> var(x) (variance)
> sd(x) (standard deviation)
```

The variance can be thought of as the *mean squared deviation (from the mean)*. If we really are interested in the average distance from the mean, then we can take the square root of the mean squared distance and thus arrive at the standard deviation.

Why the mean squared deviation? If we were to just take the average distance from the mean, we would get zero! As an exercise, you can try to show that $0 = \sum_{i=1}^n (x_i - \bar{x})$. We could instead take mean absolute deviation from the mean $|x_i - \bar{x}|$, but the absolute value function can be difficult to work with. These are two reasons why mean square deviation has become the standard.

What is so special about standard deviation? It will appear in a variety of statistical methods later on. Generally, the vast majority of your data will be within three standard deviations of the mean. This is not an absolute rule, but you will find that it is often true.

4 Graphical statistical methods

Graphical analysis. Now we will look at some graphical methods of analyzing a dataset. Graphical methods allow us to look at the entire dataset so-to-speak without actually seeing any of the numbers! They will help give us a feel for whether the data tends to cluster near a central value, if it is evenly spread out over its entire range, or if it has multiple clusters at different locations.

Stem-and-leaf plot. A stem and leaf plot consists of creating stems with the first digit of your data values, and the leaves are the remaining part of the data values, e.g. for the number 23, 2 is the stem and 3 is the leaf. This can be difficult if you have data with more than two digits or a variety of decimal places, etc.

Stem and leaf plots are useful in that they are a graphical method, but still retain some of the actual numerals in the data.

Here is the R command along with a stem plot for our water well data:

```
> stem(x)
```

```
  The decimal point is 2 digit(s) to the right of the |
```

```
0 | 7889
1 | 0001112222267788
2 | 000244566899
3 | 00068
4 | 2344456788
5 | 002689
6 | 0022446689
7 | 0
```

Notice that the `stem()` command in R rounds the data. The smallest number in the stem plot above is 0|7 with the decimal place 2 digits to the right of the “—” thus 0|7 is actually 70, but our smallest data value is actually 65. R will round all data points in order to make the leaves single digits.

If we want to make more or less leaves, we do so by setting the ‘scale:’

```
> stem(x,scale=1) (default)
```

```
> stem(x,scale=2) (more leaves)
> stem(x,scale=0.5) (less leaves)
```

Dotplot. A dotplot places a dot for each datapoint on a numberline, and successive dots for identical data points are stacked on top of each other. The dotplot command in R is a bit complicated. It is called a stacked stripchart, and we must specify to use the dot symbol with a “pch=19” option.

```
> stripchart(x,method="stack",pch=19)
```

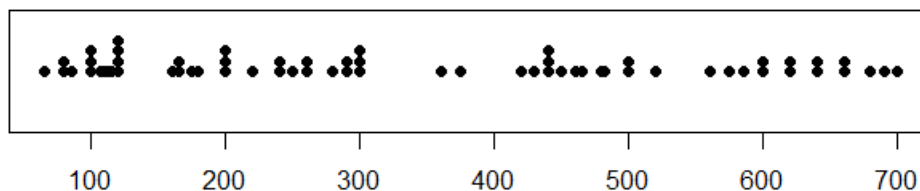


Figure 3: Dotplot of water well data.

Boxplot. A box plot is created from the five number summary. It is often called a box-and-whiskers plot as well. The box is created by the quartiles and median. The whiskers extend to the last non-outlier data point in R. Outliers are displayed as dots beyond the whiskers.

```
> boxplot(x) (this will generate a vertical boxplot)
> boxplot(x,horizontal=TRUE) (this will generate a horizontal boxplot)
```

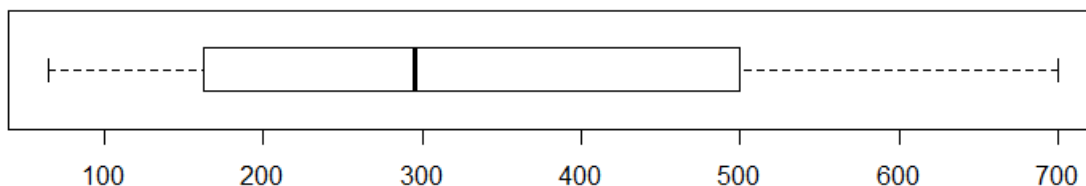


Figure 4: Boxplot of water well data.

Histogram. A histogram is probably one of the most common graphical displays of data. The data are grouped into ‘bins’ and the number of data points in each bin are counted. This creates a *frequency histogram*. There are three types of histograms. If we divide the frequencies by the sample size, then we create a *relative frequency histogram*. Relative frequencies sum to one. When we divide the relative frequencies by the width of the bin, then we create a *density histogram* which has total area one. The R histogram command makes frequency and density histograms only. A relative frequency histogram can be made, but it is a bit trickier.

frequency = number of data points in the bin (histogram bar heights sum to sample size)

relative frequency = $\frac{\text{frequency}}{\text{sample size}}$ (histogram bar heights sum to one)

density = $\frac{\text{relative frequency}}{\text{bin width}}$ (histogram bars have total area one)

> `hist(x)` (basic frequency histogram)
> `hist(x,freq=FALSE)` (basic density histogram)

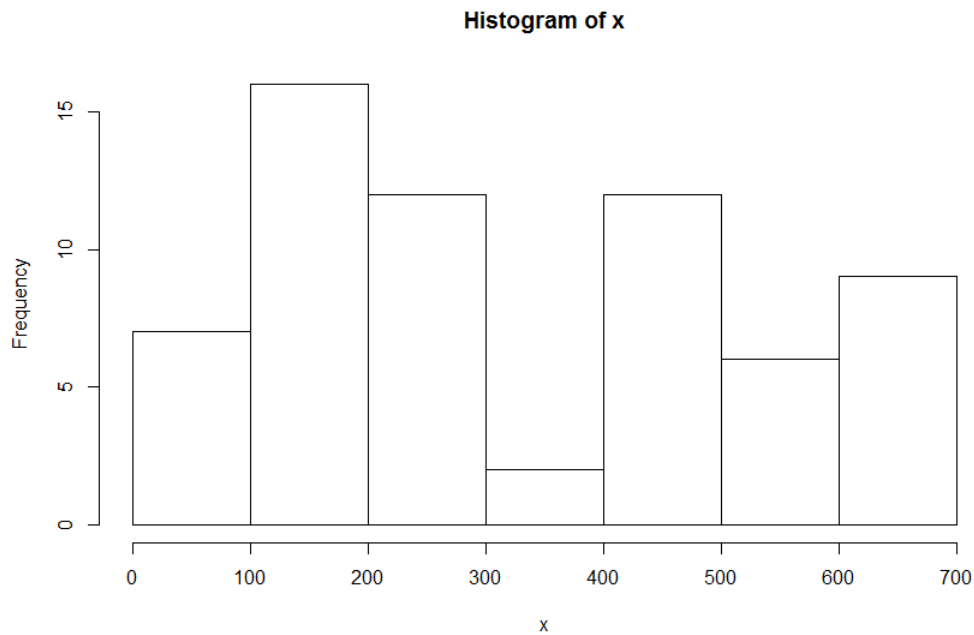


Figure 5: Frequency histogram of water well data.

It is worth knowing a bit about the structure of the R histogram object. Let's save the histogram under a name `wellhg` and then type that name and `hist` **Enter** to see all of the information and attributes it contains.

```
> wellhg = hist(x)
> wellhg
$'breaks'
[1] 0 100 200 300 400 500 600 700

$counts
[1] 7 16 12 2 12 6 9

$density
[1] 0.00109375 0.00250000 0.00187500 0.00031250 0.00187500 0.00093750 0.00140625

$mids
```

```
[1] 50 150 250 350 450 550 650
```

```
$xname
```

```
[1] "x"
```

```
$equidist
```

```
[1] TRUE
```

```
attr(,"class")
```

```
[1] "histogram"
```

Note that we can call the midpoints of our bins as

```
> wellhg$mids
```

```
[1] 50 150 250 350 450 550 650
```

We can similarly call the breaks (endpoints of bins) with `wellhg$breaks`, frequencies with `wellhg$counts`, or densities with `wellhg$density`.

Often, choosing the bins is a bit of an art. Too few bins can give a histogram that is misleading, and too many bins can give a histogram that is too jagged or doesn't sufficiently summarize the shape of the data. Figure 6 shows several different histograms for the same dataset. You can decide for yourself which is the best histogram, but I prefer the two in the middle column.

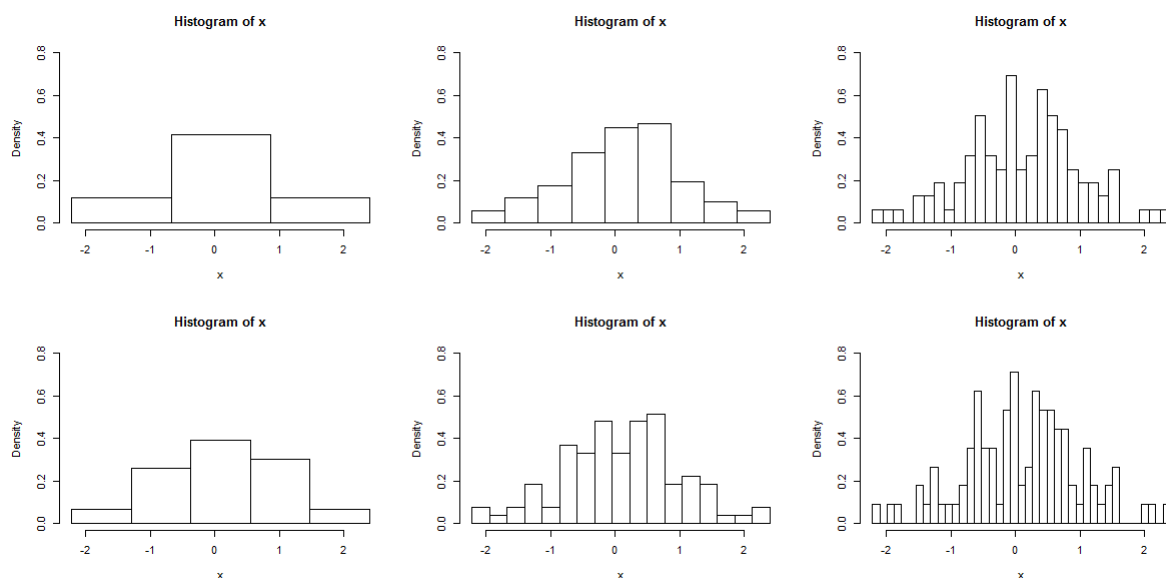


Figure 6: Histograms for the same dataset plotted with different bin sizes.

Here is the R code used to generate Figure 6:

```
> set.seed(1)
> x=rnorm(100)
> par(mfcol=c(2,3))
> nbins=3
> bins=seq(from=min(x),to=max(x),length.out=nbins+1)
```

```

hist(x,breaks=bins,freq=FALSE,ylim=c(0,0.8))
nbins=5
bins=seq(from=min(x),to=max(x),length.out=nbins+1)
hist(x,breaks=bins,freq=FALSE,ylim=c(0,0.8))
nbins=9
bins=seq(from=min(x),to=max(x),length.out=nbins+1)
hist(x,breaks=bins,freq=FALSE,ylim=c(0,0.8))
nbins=17
bins=seq(from=min(x),to=max(x),length.out=nbins+1)
hist(x,breaks=bins,freq=FALSE,ylim=c(0,0.8))
nbins=29
bins=seq(from=min(x),to=max(x),length.out=nbins+1)
hist(x,breaks=bins,freq=FALSE,ylim=c(0,0.8))
nbins=41
bins=seq(from=min(x),to=max(x),length.out=nbins+1)
hist(x,breaks=bins,freq=FALSE,ylim=c(0,0.8))

```

Here are a few different ways to create bins for histograms in R.

- > `c(b0,b1,b2,...,bm)` (creates bins $[b_0, b_1], (b_1, b_2], (b_2, b_3], \dots$)
- > `seq(from=a,to=b,by=d)` (creates bins $[a, a + d], (a + d, a + 2d], \dots$)
- > `seq(from=a,to=b,length.out=m)` (creates m total bins from a to b)