SPOILER ALERT!!!
DO NOT proceed
until you have attempted Quiz 4

MATH 259 – QUIZ 4 – SPRING 2019 || 25 POINTS || 20 MINUTES

Instructions: Show all work. No collaboration or references. No computational devices allowed without instructor permission.	Print Name: Solutions

1. (6 pts) Calculate the differential of volume for a cylindrical can with end caps: $V = \pi r^2 h$. Estimate the change in volume for a can with radius 2 inches and height 10 inches if the radius is increased by 0.1 in and the height decreased by 0.1 in.

$$dV = \frac{\partial V}{\partial r} \cdot dr + \frac{\partial V}{\partial h} \cdot dh$$
$$= 2\pi r h \cdot dr + \pi r^2 \cdot dh$$

$r=2,\ h=10$

$\Delta r = +0.1$

$\Delta h = -0.1$

$$\Delta V \approx 2\pi \cdot 2 \cdot 10 \cdot (0.1) + \pi \cdot (2)^2 \cdot (-0.1)$$
$$= 4\pi - 0.4\pi$$
$$= \boxed{3.6\pi} \approx 3.6 \times 3 \approx 10 \text{ to } 12$$

so abt. 10 or so cubic inch increase in volume

*(Note: Actual $\Delta z \cong 11.5$) so this is a good estimate!

2. (5 pts) Find the derivative of $f(x, y) = x^2y + xy^3$ at the point $(2, 1)$ in direction $\langle -1, 1\rangle$.

$$\nabla f = \langle 2xy + y^3,\ x^2 + 3xy^2 \rangle$$

$$\nabla f(2,1) = \langle 2\cdot 2\cdot 1 + 1^3,\ 2^2 + 3\cdot 2\cdot 1^2 \rangle$$
$$= \langle 5, 10 \rangle$$

$$D_{\vec{v}} f = \frac{\nabla f \cdot \vec{v}}{|\vec{v}|} = \frac{\langle 5,10\rangle \cdot \langle -1,1\rangle}{\sqrt{2}}$$

$$= \frac{-5+10}{\sqrt{2}} = \boxed{\frac{5}{\sqrt{2}}} \approx \frac{5}{1.4} \approx 3.5 \text{ or so}$$

$1.4 \times 1.4 \times 1.4$

1.4
1.4
1.4
$\overline{4.2}$

3. (4 pts) Use the chain rule to find $\frac{\partial z}{\partial t}$ for $z = e^{-x^2}\sin(y)$ where $x = s^2t$ and $y = s + t^3$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= -2xe^{-x^2}\cdot\sin(y)\cdot s^2 + e^{-x^2}\cdot\cos(y)\cdot 3t^2$$

4. (6 pts) Find the equation of the tangent plane to the surface given by $f(x, y) = x^2y + xy^3$ at the point $(2, 1)$.

normal is $\langle \nabla f, -1\rangle$ or $\langle f_x, f_y, -1\rangle$

$f(2,1) = 2^2\cdot 1 + 2\cdot 1^3 = 6$

so. pt. on plane: $\underset{x_0\ y_0\ z_0}{(2, 1, 6)}$

& normal vector is $\langle 5, 10, -1\rangle$ (from previous page...)

eq. of plane: $\boxed{5(x-2) + 10(y-1) - (z-6) = 0}$

5. (4 pts) Find all critical points and local extrema of $f(x, y) = x^2 + y^2 - xy$.
(Note: Part of the formulas for the second derivative are given.)

$D = f_{xx}f_{yy} - (f_{xy})^2$
$D > 0$ $f_{xx} > 0$ min
$D > 0$ $f_{xx} < 0$ max
$D < 0$ ~~inconclusive~~ saddle
$D = 0$ inconclusive

$f_x = 2x - y \to f_x = 0 \Rightarrow 2x = y$

$f_y = 2y - x \to f_y = 0 \Rightarrow 2y = x$

$f_{xx} = 2$
$f_{yy} = 2$
$f_{xy} = -1$

$D = 2\cdot 2 - (-1)^2$
$= 3$

$2x = y$ & $2y = x$

$+2(\ \ )$

$4x = 2y = x$
$\Rightarrow 4x = x$
$\Rightarrow x = 0$
$\Rightarrow y = 0$

Crit. pt. $(0,0)$

$f_{xx} = 2 > 0$

$D = 2 > 0$

$\Rightarrow$ local min.

(Note:
$f(x,y) = \underbrace{x^2 + y^2}_{\text{like a parabola}} - xy$
like a parabola & this grows slower than x^2+y^2.)