



SPOILER ALERT!!!
DO NOT proceed
until you have attempted Quiz 4

Instructions: Show all work. No collaboration or references.
No computational devices allowed without instructor permission.

Print Name

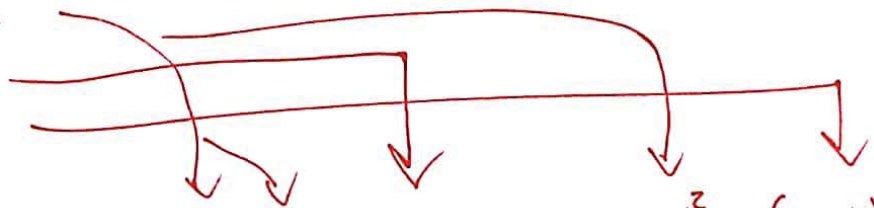
Solutions

1. (6 pts) Calculate the differential of volume for a cylindrical can with end caps: $V = \pi r^2 h$. Estimate the change in volume for a can with radius 2 inches and height 10 inches if the radius is increased by 0.1 in and the height decreased by 0.1 in.

$$dV = \frac{\partial V}{\partial r} \cdot dr + \frac{\partial V}{\partial h} \cdot dh$$

$$= 2\pi r h \cdot dr + \pi r^2 \cdot dh$$

$r=2, h=10$
 $\Delta r = +0.1$
 $\Delta h = -0.1$



$$\Delta V \approx 2\pi \cdot 2 \cdot 10 \cdot (0.1) + \pi \cdot (2)^2 \cdot (-0.1)$$

$$= 4\pi - 0.4\pi$$

$$= \boxed{3.6\pi} \approx 3.6 \times 3 \approx 10 + 0.2$$

so abt. 10 or so
cubic inch increase
in volume

*(Note: Actual $\Delta z \approx 11.5$) so this is a good estimate!

2. (5 pts) Find the derivative of $f(x, y) = x^2 y + xy^3$ at the point (2, 1) in direction $\langle -1, 1 \rangle$.

$$\nabla f = \langle 2xy + y^3, x^2 + 3xy^2 \rangle$$

$$\nabla f(2, 1) = \langle 2 \cdot 2 \cdot 1 + 1^3, 2^2 + 3 \cdot 2 \cdot 1^2 \rangle$$

$$= \langle 5, 10 \rangle$$

$$D_{\vec{v}} f = \frac{\nabla f \cdot \vec{v}}{|\vec{v}|} = \frac{\langle 5, 10 \rangle \cdot \langle -1, 1 \rangle}{\sqrt{2}}$$

$$= \frac{-5 + 10}{\sqrt{2}} = \boxed{\frac{5}{\sqrt{2}}} \approx \frac{5}{1.4} \approx 3.5 \text{ or so}$$

$\begin{matrix} 1.4 \\ 1.4 \\ 1.4 \\ \hline 4.2 \end{matrix}$

3. (4 pts) Use the chain rule to find $\frac{\partial z}{\partial t}$ for $z = e^{-x^2} \sin(y)$ where $x = s^2t$ and $y = s + t^3$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$= (-2x e^{-x^2} \cdot \sin(y)) \cdot 2s^2 + e^{-x^2} \cdot \cos(y) \cdot 3t^2$$

4. (6 pts) Find the equation of the tangent plane to the surface given by $f(x, y) = x^2y + xy^3$ at the point $(2, 1)$.

normal is $\langle \nabla f, -1 \rangle$ or $\langle f_x, f_y, -1 \rangle$

$$f(2, 1) = 2^2 \cdot 1 + 2 \cdot 1^3 = 6$$

so, pt. on plane: $(2, 1, 6)$

& normal vector is $\langle 5, 10, -1 \rangle$ (from previous page...)

eq. of plane:

$$5(x-2) + 10(y-1) - (z-6) = 0$$

5. (4 pts) Find all critical points and local extrema of $f(x, y) = x^2 + y^2 - xy$.

(Note: Part of the formulas for the second derivative are given.)

$D = f_{xx}f_{yy} - (f_{xy})^2$
 $D > 0$ $f_{xx} > 0$ min
 $D > 0$ $f_{xx} < 0$ max
 $D < 0$ ~~inconclusive~~ saddle
 $D = 0$ inconclusive

crit. pt. $(0, 0)$

$$f_{xx} = 2 > 0$$

$$D = 2 > 0$$

\Rightarrow local min.

$$f_x = 2x - y \rightarrow f_x = 0 \Rightarrow 2x = y$$

$$f_y = 2y - x \rightarrow f_y = 0 \Rightarrow 2y = x$$

$$\left. \begin{aligned} f_{xx} &= 2 \\ f_{yy} &= 2 \\ f_{xy} &= -1 \end{aligned} \right\}$$

$$D = 2 \cdot 2 - (-1)^2 = 3$$

$$2x = y \text{ \& } 2y = x$$

$$\begin{aligned} +2(\quad) \\ 4x = 2y = x \end{aligned}$$

$$\Rightarrow 4x = x$$

$$\Rightarrow x = 0$$

$$\Rightarrow y = 0$$

Note: $f(x, y) = x^2 + y^2 - xy$
 like a parabola & this grows slower than $x^2 + y^2$