FORMULAS:

LINES & PLANES

$\mathbf{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct\rangle$ $\quad$ $\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1$ $\quad$ $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

VECTOR PRODUCTS

$\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\cos\theta$ $\quad$ $|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta$

PARALLELOGRAM & PARALLELEPIPED

$A = |\mathbf{a}\times\mathbf{b}|$ $\quad$ $V = |\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$

CURVATURE

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

PROJECTIONS

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|}\right)\frac{\mathbf{b}}{|\mathbf{b}|} \qquad \text{comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|}$$

DIRECTIONAL DERIVATIVE

$$D_{\mathbf{v}}f = \nabla f\cdot\frac{\mathbf{v}}{|\mathbf{v}|}$$

$$\nabla = \langle \tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z}\rangle$$

VECTOR PRODUCTS

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1\rangle$$

$$\mathbf{u}\cdot\mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$$

LINE INTEGRALS

$$\int_C f(x,y)\ ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2 + (y'(t))^2}\ dt \qquad \int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \mathbf{F}\cdot\mathbf{T}\ ds = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\ dt$$

ARC LENGTH, SURFACE AREA & VOLUME

$$L(C) = \int_C ds = \int_a^b ||\mathbf{r}'(t))||\ dt \qquad A(S) = \iint_S dS = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\ dA = \iint_D |\mathbf{r}_u\times\mathbf{r}_v|\ dA \qquad V(E) = \iiint_E dV$$

CURL AND DIVERGENCE

$$\text{curl } \mathbf{F} = \nabla\times\mathbf{F} = \langle R_y - Q_z, -(R_x - P_z), Q_x - P_y\rangle \qquad \text{div } \mathbf{F} = \nabla\cdot\mathbf{F} = P_x + Q_y + R_z$$

SCALAR SURFACE INTEGRALS

$$\iint_S f(x,y,z)\ dS = \iint_D f(\mathbf{r}(u,v))\,|\mathbf{r}_u\times\mathbf{r}_v|\ dA$$

VECTOR SURFACE INTEGRALS (FLUX)

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S \mathbf{F}\cdot\mathbf{n}\ dS = \iint_D \mathbf{F}\cdot\mathbf{n}\,|\mathbf{r}_u\times\mathbf{r}_v|\ dA = \iint_D \mathbf{F}\cdot(\mathbf{r}_u\times\mathbf{r}_v)\ dA$$

$\mathbf{n} = \dfrac{\mathbf{r}_u\times\mathbf{r}_v}{|\mathbf{r}_u\times\mathbf{r}_v|}$ for a parametrized surface $\qquad$ $\mathbf{n} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$ where $\mathbf{v} = \langle -g_x, -g_y, 1\rangle$ for a surface $z = g(x,y)$

GREEN'S THEOREM, CIRCULATION AND FLUX FORMS

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \boxed{\oint_C P\ dx + Q\ dy = \iint_D (Q_x - P_y)\,dA} = \iint_D (\text{curl } \mathbf{F})\cdot\mathbf{k}\ dA \qquad \oint_C \mathbf{F}\cdot\mathbf{n}\ ds = \iint_D (P_x + Q_y)\ dA$$

STOKE'S THEOREM

$$\boxed{\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F}\cdot d\mathbf{S}} = \iint_D \text{curl } \mathbf{F}\cdot(\mathbf{r}_u\times\mathbf{r}_v)\ dA$$

DIVERGENCE THEOREM

$$\boxed{\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F}\ dV}$$