Math 321 – Summer 2019

additional practice for chapters 2-3, probability theory and distributions

1. A batch of fuses produced by a particular factory are tested. There are two types of fuses. 55% of the fuses are type I, and 45% are type II. 5% of the type I fuses fail to meet quality specifications, and 4% of type II fail.
   (a) What is the probability a type I fuse satisfies quality specifications?
   (b) If a fuse is randomly selected, what is the probability it is of type I and satisfied quality specs?
   (c) What is the probability a randomly selected fuse satisfies quality specs?
   (d) Given that we know a particular fuse has failed to meet quality specs, what is the probability it is type I?
   (e) Are fuse type and whether it passes quality specs independent?

2. We roll a pair of 6-sided dice. Let $A = \{\text{sum is even}\}$, and $B = \{\text{both dice are even}\}$. Calculate $P(A)$, $P(B)$, $P(A \mid B)$, and $P(B \mid A)$. Are A and B independent?

3. Now let's roll a single 6-sided die. Let events: $A = \{2, 4, 6\} = \{\text{even}\}$, and $B = \{1, 2\}$, $C = \{1, 2, 3\}$, $D = \{1, 2, 3, 4\}$. Is A independent of any of the other 3 events?

4. A particular political candidate is running for office. It is known that 20% of party D voters support the candidate, that 60% of party R voters support the candidate, and that 40% of independent (no party affiliation) voters support the candidate. The general population of voters is made up of equal proportions of each party but 20% of the population are independents. Given that a randomly selected voter supports the candidate, what is the probability they are a member of party D? Are political party membership and support for the candidate independent events?

5. If you flip three fair coins, and there is at least one head, what is the probability that there are at least two heads?

6. Let's look at flipping two fair coins. Show that the coins are *independent*.

7. Consider the probability mass function given below.

x	1	2	3	4
$f(x)$	0.4	0.3	0.2	0.1

   a) Calculate the mean.
   b) Calculate the variance.
   c) Calculate $E(10 - 3X)$.

8. A production line produces items which are defective with probability 2%. Suppose 1,500 items are produced during a day.
   a) Write the probability that x items are defective.
   b) Calculate the probability that at least 50 items are defective.
   c) Calculate the probability that at least 20 but no more than 40 items are defective.

9. Flip a fair coin 10 times. Calculate the probability that you get either 2 or 3 heads.

10. Suppose a storage closet contains 25 rock samples, 17 of them are volcanic in origin and 8 of them are sedimentary. If a geologist selects 10 rock samples randomly, what is the probability that x of them are sedimentary?

11. Suppose a manufacturer produces PVC pipe that has on average 3 defects per 100 ft of pipe. If a wastewater construction agency purchases 250 ft of pipe, what is the probability that it will have a total of less than 3 defects?

12. Flip a biased coin with probability of heads being 0.2. Refer to getting heads as a success.

    (a) If we flip the coin 20 times, what is the probability of getting 13 heads?

    (b) Now, let's flip the coin until the first head occurs. Write the probability mass function for $X = \{\text{the total number of flips up to and including the first head}\}$. What is the probability that there are 5 tails before the first head?

13. Consider a population of animals where adult weight is normally distributed with mean 7.3 kg and standard deviation 1.8 kg.

    (a) If an individual is randomly selected, what is the probability its weight is less than 5 kg?

    (b) What is the probability that a randomly selected individual's weight will be between 8.2 and 9.5 kg?

    (c) What is the probability that a randomly selected individual's weight will be between 5.5 and 9.1 kg?

    (d) What is the probability that a randomly selected individual's weight will be above 10.6 kg?

    (e) If a sample of 12 individuals is taken, what is the probability the mean weight will be less than 5 kg?

    (f) What is the probability that the mean weight is exactly 5 kg?

    (g) If an iid sample of 25 individuals is selected, what is the probability that every individual's weight is in the range of 5 to 10 kg?

    (h) What is the probability that $\frac{X_3-9}{1.8} < -1$?

14. Consider a manufacturer producing a thin laminate material for protective coatings of surfaces. The laminate material is produced continually with a width of 1 m and rolled onto spools that hold a total length of 200 m. Assume it is known that on average there are 3 imperfections per 40 m^2.

    (a) If a particular customer is to buy a complete spool of this material, what is the probability that the total number of imperfections is greater than 25?

    (b) If that customer needs at least four 1 m by 10 m sections with no imperfections (a total of 1 m by 40 m), what is the probability that the first 40 m on the spool have no imperfections?

    (c) If a customer shows up at the laminate manufacturers and needs a 1 m by 20 m section with no imperfections, what is the probability that such a section will be produced?

15. Consider the pdf $f(x) = a(16 - x^2)$ for $0 \leq x \leq 4$.

    (a) Find a.

    (b) Find the cdf.

    (c) Find $E(X)$.

16. Consider the pdf $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. Note that this is the exponential pdf.

    (a) Find the median.

    (b) Find the first quartile.

17. Consider the pmf

x	0	10	20
$f(x)$	0.10	0.60	0.30

    (a) Calculate $E(X)$ and $Var(X)$.

    (b) Calculate $E(100 + 2X)$ and $Var(100 + 2X)$.

(c) Write the formula for and sketch a graph of the cdf.

18. Consider the uniform random variable $X \sim U(0, 100)$.

   (a) Write the pdf.
   (b) Write the cdf.
   (c) Find $E(X)$.
   (d) Find $P(50 < X < 70)$.
   (e) Find b so that $P(30 < X < b) = 0.6$.
   (f) Find the first quartile.

19. Explain the following R code.

```
> x=c(0,2,4,6,8,10)
  p=c(0.69,0.01,0.05,0.12,0.03,0.10)
  m=sum(x*p)
  v=sum(x^2*p)-m^2
```

20. Explain the following R code.

```
> x=rnorm(100,mean=25,sd=5)
  hist(x,breaks=seq(from=0,to=50,by=2))
```