

**BONUS** (10 bonus pts max) Choose one of the following three bonus problems to attempt.

I. Consider a 12-sided die where each face is marked with a number (1,2,3,4) and color (R,G,B). Thus the die faces are:  $\{1R, 2R, 3R, 4R, 1G, 2G, 3G, 4G, 1B, 2B, 3B, 4B\}$ . Assume that when rolled, each face is equally likely. When rolling 5 such dice, what is the probability of a one-color numerical pair and a tri-color numerical triple? E.g.  $3R, 3R, 5R, 5G, 5B$  or  $1G, 1G, 4R, 4G, 4B$ .

II. Consider random triple  $(X, Y, Z)$  that is uniformly distributed on the unit cube  $[0, 1]^3$ . Calculate  $P(X + Y + Z < 1)$ .

III. Consider the joint pdf  $f_{X,Y,Z}(x, y, z) = \frac{1}{12}(xy + z)$  on  $[0, 1] \times [0, 2] \times [0, 3]$ . Calculate the probability  $P(X < 1/2 \mid Y > 1, Z = 2)$ .