2.21 $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = P(B) \rightarrow \dfrac{P(A \cap B)}{P(B)} = P(A) \rightarrow P(A|B) = P(A)$

2.22 **(a)** $P(B) = P(A \cap B) + P(A' \cap B) = P(A)P(B) + P(A' \cap B)$

$P(A' \cap B) = P(B) - P(A)P(B) = P(B)[(1 - P(A)] = P(B)P(A')$ QED

(b) $P(B') = P(A \cap B') + P(A' \cap B') = P(A' \cap B') + P(B')P(A)$

$P(A' \cap B') = P(B') - P(B')P(A|B') = P(B')[(1 - P(A)] = P(B')P(A')$ QED

2.24 $P(A) = 0.60,\ P(B) = 0.80,\ P(C) = 0.50,\ P(A \cap B) = 0.48,\ P(A \cap C) = 0.30$

$P(B \cap C) = 0.38,\ P(A \cap B \cap C) = 0.24$

$P(A \cap B \cap C) = 0.24,\ P(A)(B)(C) = (0.6)(0.8)(0.5) = 0.24$

$P(B \cap C) = 0.38,\ P(B)P(C) = (0.8)(0.5) = 0.40$ B and C *not independent*

2.76 **(a)** $\dfrac{18+36}{90} = \dfrac{54}{90} = \dfrac{3}{5}$; **(b)** $\dfrac{36+27}{90} = \dfrac{63}{90} = \dfrac{7}{10}$; **(c)** $\dfrac{18}{90} = \dfrac{2}{10} = \dfrac{1}{5}$;

(d) $\dfrac{27}{90} = \dfrac{3}{10}$; **(e)** $\dfrac{18}{18+36} = \dfrac{18}{54} = \dfrac{1}{3}$; **(f)** $\dfrac{27}{27+36} = \dfrac{27}{63} = \dfrac{3}{7}$

2.94 A even first, B even second, C same number both

$P(A) = \dfrac{1}{2},\ P(B) = \dfrac{1}{2},\ P(C) = \dfrac{1}{6},\ P(A \cap B) = \dfrac{1}{4},\ P(A \cap C) = \dfrac{3}{36} = \dfrac{1}{12}$

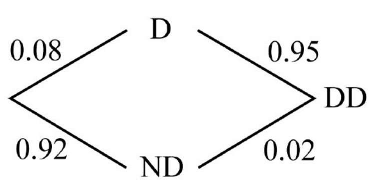
11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36,
41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66

$P(B \cap C) = \dfrac{1}{12},\ P(A \cap B \cap C) = \dfrac{3}{36} = \dfrac{1}{12} \neq \dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{6} = \dfrac{1}{24}$

(a) Since $\dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4},\ \dfrac{1}{2} \cdot \dfrac{1}{6} = \dfrac{1}{12}$, and $\dfrac{1}{2} \cdot \dfrac{1}{6} = \dfrac{1}{12}$, events are pairwise independent.

(b) Since $\dfrac{1}{2} \cdot \dfrac{1}{2} \cdot \dfrac{1}{6} \neq \dfrac{1}{12}$ the events are *not* independent.

2.108



(a) $(0.08)(0.95) + (0.92)(0.02)$

$= 0.076 + 0.0814 = 0.0944$

(b) $\dfrac{0.076}{0.0944} = 0.8051$

3.2 **(a)** No, because $f(1)$ is negative; **(b)** Yes; **(c)** No, because $f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$ is greater than 1.

3.4 **(a)** $c(1+2+3+\ldots 5)=1$; thus $C=\frac{1}{15}$

(b) $c\left(5+\frac{5}{2}+\frac{5}{3}+\frac{5}{4}+1\right)=1$; thus, $c=\frac{12}{137}$

(c) $\sum_{x=1}^{k} f(x)=c\sum_{x=1}^{k} x^2 = cS(k,2)$

From Theorem A.1 we obtain $S(k,2)=\frac{1}{6}k(k+1)(2k+1)$

Thus, for $f(x)$ to be a distribution function, $c=\frac{6}{k(k+1)(2k+1)}$, $k \neq 0$.

(d) $\sum_{x=1}^{\infty} f(x)=c\sum_{x=1}^{\infty}\left(\frac{1}{4}\right)^x$

The right-hand sum is a geometric progression with a = 1 and r = 1/4. For x = 1 to n, this sum equals

$$S_n=\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}}\rightarrow\frac{1/4}{3/4}=\frac{1}{3}$$ as $n\rightarrow\infty$. Therefore, $c=3$.

3.13 **(a)** $\frac{3}{4}$ **(b)** $\frac{3}{4}-\frac{1}{2}=\frac{1}{4}$ **(c)** $\frac{1}{2}$ **(d)** $1-\frac{1}{4}=\frac{3}{4}$

(e) $\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$ **(f)** $1-\frac{3}{4}=\frac{1}{4}$

3.20 **(a)** $\int_2^{3.2}\frac{1}{8}(y+1)dy=\frac{1}{8}\left(\frac{y^z}{2}+y\right)\Bigg|_2^{3.2}=\frac{1}{8}(8.32-4)=0.54$

(b) $\int_{2.9}^{3.2}\frac{1}{8}(y+1)dy=\frac{1}{8}\left(\frac{y^x}{2}+y\right)\Bigg|_{2.9}^{3.2}=\frac{1}{8}(8.32-7.105)=0.1519$

3.24 $F(z)=k\int_0^z ze^{-z^z}dz=k\int_0^z\frac{1}{2}e^{-u}du=\frac{k}{2}[1-e^{-u}]=\frac{k}{2}(1-e^{-z^z})$ $k=2$

3.34 **(a)** $F(5)=1-\frac{9}{25}=\frac{16}{25}$

(b) $1-F(8)=1-1+\frac{9}{64}=\frac{9}{64}$