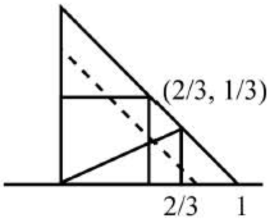
3.42 **(a)** $\frac{1}{20}$; **(b)** $\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$; **(c)** $\frac{1}{6}+\frac{1}{4}+\frac{1}{12}=\frac{1}{2}$; **(d)** $\frac{1}{6}+\frac{1}{24}+\frac{1}{40}=\frac{28}{120}=\frac{7}{30}$

3.43 **(a)** $\frac{1}{6}+\frac{1}{12}=\frac{1}{4}$; **(b)** 0; **(c)** $\frac{1}{12}+\frac{1}{6}+\frac{1}{24}=\frac{7}{24}$; **(d)** $1-\frac{1}{120}=\frac{119}{120}$

3.51



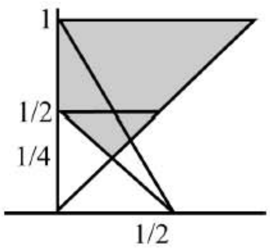
(a) $\frac{1}{2}$

(b) $1-2\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{2}{3}=1-\frac{4}{9}=\frac{5}{9}$

(c) $2\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{3}\right)=\frac{3}{9}=\frac{1}{3}$

$F(x,y)=2xy$ for $x>0,\ y>0,\ x+y<1$

3.53



$$\int_{1/4}^{1/2}\frac{1}{y}\int_{1/2-y}^{y}dx\,dy+\int_{1/2}^{1}\frac{1}{y}\int_{0}^{y}dx\,dy$$

$$=1-\frac{1}{2}\ln 2=1-0.3466=0.6534$$

3.56 $\frac{\partial F}{\partial x}=e^{-x}-e^{-x-y}\ \frac{\partial^2 F}{\partial x\partial y}=e^{-x-y}\quad x>0,\ y>0$

$= 0$ elsewhere

3.68 **(a)** $\frac{1}{3}\int_0^{1/2}\int_0^{1/2}\int_0^{1/2}(2x+3y+z)\,dz\,dy\,dx$

$$=\frac{1}{3}\int_0^{1/2}\int_0^{1/2}\left[(2x+3y)z+\frac{z^2}{2}\right]_0^{1/2}dy\,dx$$

$$=\frac{1}{3}\int_0^{1/2}\int_0^{1/2}\left(x+\frac{3}{2}y+\frac{1}{8}\right)dy\,dx$$

$$=\frac{1}{3}\int_0^{1/2}\left(xy+\frac{3}{4}y^2+\frac{1}{8}y\right)\Bigg|_0^{1/2}dx=\frac{1}{3}\int_0^{1/2}\left(\frac{1}{2}x+\frac{3}{16}+\frac{1}{16}\right)dx$$

$$=\frac{1}{3}\left(\frac{1}{16}+\frac{3}{32}+\frac{1}{32}\right)=\frac{1}{3}\cdot\frac{6}{32}=\frac{1}{16}$$

3.69 **(a)** $g(-1)=\frac{1}{4},\ g(1)=\frac{3}{4}$

(b) $h(-1)=\frac{5}{8},\ h(0)=\frac{1}{4},\quad h(1)=\frac{1}{8}$

(c) $f(-1|-1)=\frac{1/8}{1/8+1/2}=\frac{1}{5};\quad f(1|-1)=\frac{1/2}{1/8+1/2}=\frac{4}{5}$

3.71 **(a)** $m(x,y)=\dfrac{xy}{108}(1+2)=\dfrac{xy}{36}$ for $x=1,2,3$; $y=1,2,3$

(b) $n(x,z)=\dfrac{xz}{108}(1+2+3)=\dfrac{xz}{18}$ for $x=1,2,3$; $z=1,2$

(c) $g(x)=\dfrac{x}{36}(1+2+3)=\dfrac{x}{6}$ for $x=1,2,3$

(d) $\phi(z|1,2)=\dfrac{z/64}{2/36}=\dfrac{z}{3}$ for $z=1,2$

(e) $\psi(y,z|3)=\dfrac{yz/36}{1/2}=\dfrac{yz}{18}$ for $y=1,2,3$; $z=1,2$

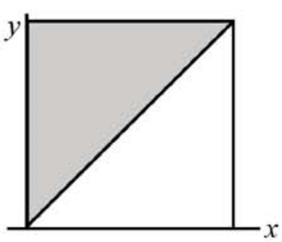
3.74 **(a)** $\dfrac{1}{4}\int_0^2(2x+y)\,dy=\dfrac{1}{4}\left[2xy+\dfrac{y^2}{2}\right]\Bigg|_0^2=\dfrac{1}{4}(4x+2)=\dfrac{1}{2}(2x+1)$ for $0<x<1$

$= 0$ elsewhere

(b) $f\left(y\middle|\dfrac{1}{4}\right)=\dfrac{\dfrac{1}{4}\left(\dfrac{1}{2}+y\right)}{\dfrac{1}{2}\cdot\dfrac{3}{2}}=\dfrac{1}{6}(2y+1)$ for $0<y<2$

$= 0$ elsewhere

3.77



(a) $g(x)=\int_x^1\dfrac{1}{y}dy=\ln y\Big|_x^1=\ln 1-\ln x=\begin{cases}-\ln x & 0<x<1\\ 0 & \text{elsewhere}\end{cases}$

(b) $h(y)=\int_0^y\dfrac{1}{y}dx=\dfrac{1}{y}(y-0)=\begin{cases}1 & 0<y<1\\ 0 & \text{elsewhere}\end{cases}$

3.109 **(a)** $f(x_1,x_2,x_3)=\begin{cases}\dfrac{(20{,}000)^3}{(x_2+100)^3(x_2+100)^3(x_3+100)^3} & x_1>0,\ x_2>0,\ x_3>0\\ 0 & \text{elsewhere}\end{cases}$

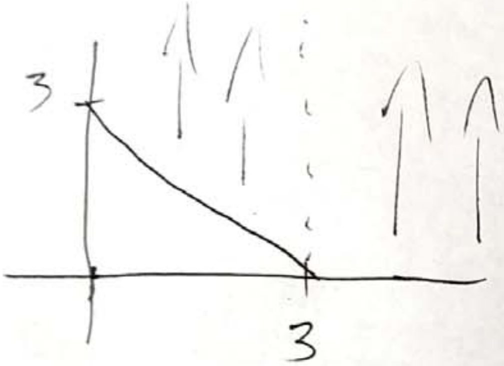
(b) $\int_0^{100}\dfrac{20{,}000}{(x_1+100)^3}dx_1\int_0^{100}\dfrac{20{,}000}{(x_2+100)^3}dx_2\int_{200}^{\infty}\dfrac{20{,}000}{(x_3+100)^3}dx_3$

$=\dfrac{3}{4}\cdot\dfrac{3}{4}\cdot\dfrac{1}{9}=\dfrac{1}{16}$

3.57) $P(X+Y>3) = P(Y>3-x)$

$f(x,y) = e^{-x}e^{-y} \quad x>0, y>0$

$$= \int_0^3 \int_{3-x}^{\infty} e^{-x} e^{-y}\, dy\, dx + \int_3^{\infty} \int_0^{\infty} e^{-x} e^{-y}\, dy\, dx$$



$$= \int_0^3 e^{-x}\left[-e^{-y}\right]_{3-x}^{\infty} dx + \int_3^{\infty} e^{-x}\left[-e^{-y}\right]_0^{\infty} dx$$

$$= \int_0^3 e^{-x} \cdot e^{-(3-x)}\, dx + \int_3^{\infty} e^{-x}\, dx$$

$$= \int_0^3 e^{-3}\, dx + \left[-e^{-x}\right]_3^{\infty}$$

$$= 3 \cdot e^{-3} + e^{-3}$$

$$= \boxed{4e^{-3}}$$