4.5 **(a)** $E(x)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\,dy\,dx$; $E(x)=\int_{-\infty}^{\infty} x\, g(x)\,dx$

4.6 $E(x)=(-1)\left(\frac{3}{7}\right)+0\left(\frac{2}{7}\right)+1\left(\frac{1}{7}\right)+3\left(\frac{1}{7}\right)=\frac{1}{7}$

4.7 $E(Y)=\frac{1}{8}\int_{2}^{4}(y^2+y)\,dy=\frac{1}{8}\left[\frac{y^3}{3}+\frac{y^2}{2}\right]_2^4=\frac{1}{8}\left(\frac{64}{3}+8-\frac{8}{3}-2\right)$

$$=\frac{1}{8}\left(\frac{56}{3}+6\right)=\frac{1}{8}\cdot\frac{74}{3}=\frac{37}{12}$$

4.8 $E(x)=\int_{0}^{1}x^2\,dx+\int_{1}^{2}(2x-x^2)\,dx=\frac{1}{3}+\left[x^2-\frac{x^2}{3}\right]_1^2=\frac{1}{3}+4-\frac{8}{3}-1+\frac{1}{3}$

$$=3-\frac{6}{3}=1$$

4.12 $E(x)=2,\quad E(Y)=\frac{19}{15}$, and $E(2x-Y)=\frac{60-19}{15}=\frac{41}{15}=2\frac{11}{15}$

Marginal distributions

x	0	1	2	3
$g(x)$	1/10	1/5	3/10	2/5

y	0	1	2
$h(y)$	1/5	1/3	14/30

$E(x)=\frac{1}{5}+\frac{6}{10}+\frac{6}{5}=\frac{20}{10}=2$

$E(Y)=\frac{1}{3}+\frac{28}{30}=\frac{38}{30}=\frac{19}{15}$

4.13 $E\left(\frac{x}{y}\right)=\int_0^1\int_0^y\frac{x}{y^2}\,dx\,dy=\int\frac{1}{2}\,dy=\frac{1}{2}$

4.14 $k=\frac{1}{54}$

for x $\quad g(1)=\frac{1}{54}(1+2+2+4+3+6)=\frac{18}{54}=\frac{1}{3}$

$g(2)=\frac{2}{3}$

for y $\quad h(1)=\frac{1}{54}(1+2+2+4)=\frac{1}{6};\quad h(2)=\frac{1}{3},\quad h(3)=\frac{1}{2}$

for z $\quad \phi(1)=\frac{1}{3};\quad \phi(2)=\frac{2}{3}$

$E(x)=1\cdot\frac{1}{3}+2\cdot\frac{2}{3}=\frac{5}{3},\quad E(Y)=1\cdot\frac{1}{6}+2\cdot\frac{1}{3}+3\cdot\frac{1}{2}=\frac{1+4+9}{6}=\frac{14}{6}=\frac{7}{3}$

$E(z)=\frac{5}{3},\quad E(u)=\frac{5}{3}+\frac{7}{3}+\frac{5}{3}=\frac{17}{3}$

4.19 $\mu = \int_0^2 \frac{x^2}{2} dx = \frac{x^3}{6}\Big|_0^2 = \frac{4}{3}$, $\mu_2' = \int_0^2 \frac{x^3}{2} dx = \frac{x^4}{8}\Big|_0^2 = 2$

$\sigma^2 = 2 - \frac{16}{9} = \frac{2}{9}$

4.22 $\text{var}(2x+3) = 4\ \text{var}(x)$

$\mu = 1$ from Exercise 4.8

$\mu_2' = \int_0^1 x^3 dx + \int_1^2 (2x^2 - x^3) dx = \frac{1}{4} + \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]\Big|_1^2$

$= \frac{1}{4} - \frac{16}{3} - 4 - \frac{2}{3} + \frac{1}{4} = \frac{7}{6}$, $\sigma^2 = \frac{7}{6} - 1 = \frac{1}{6}$

$\text{var}(2x+3) = 4 \cdot \frac{1}{6} = \frac{2}{3}$

4.23 $E(z) = E\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma}E(x-\mu) = \frac{1}{\sigma}(\mu - \mu) = 0$ exists

$\text{var}(z) = E\left[\left(\frac{x-\mu}{\sigma}\right)^2\right] = \frac{1}{\sigma^2}E[(x-\mu)^2] = \frac{\sigma^2}{\sigma^2} = 1$

4.24 $E(x) = \int_1^\infty 2x^{-2} dx = [-2x^{-1}] = \frac{2}{1} = 2$ exists

$\mu_2' = \int_1^\infty \frac{2}{x} dx = 2\ln x\Big|_1^\infty = \infty$ σ^2 does *not* exist

4.29 $\mu = \int_0^a xf(x)\, dx + \int_a^\infty xf(x)\, dx \ge a\int_a^\infty f(x)\, dx = aP(x \ge a)$

$\frac{\mu}{a} \ge P(x \ge a)$ QED

4.31 **(a)** $1 - \frac{1}{k^2} = 0.95$, $\frac{1}{k^2} = 0.05 = \frac{1}{20}$, $k = \sqrt{20} = 4.47$

(b) $1 - \frac{1}{k^2} = 0.99$, $\frac{1}{k^2} = 0.01 = \frac{1}{100}$, $k = 10$

4.33 $M_x(t) = \int_0^t \left(e^{tx}\, dx\right) = \frac{e^{tx}}{t}\Big|_0^1 = \frac{e^t - 1}{t}$

$\frac{e^t - 1}{t} = 1 + \frac{t}{2!} + \frac{t^2}{3!} + \ldots \mu_1' = \frac{1}{2}$ and $\mu_2' = \frac{1}{3}$

$\sigma^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

4.34 $M_x(t) = \sum 2\left(\frac{1}{3}\right)^x e^{tx} = \sum_1^\infty 2\left(\frac{e^t}{3}\right)^x = \frac{2\left(\frac{e^t}{3}\right)}{1-\left(\frac{e^t}{3}\right)} = \frac{2e^t}{3-e^t}$

$$M'(t) = \frac{(3-e^t)2e^t - 2e^t(-e^t)}{(3-e^t)^2} = \frac{6e^t}{(3-e^t)^2}$$

$$M''(t) = \frac{(3-e^t)6e^t - 6e^t \cdot 2(3-e^t)(-e^t)}{(3-e^t)^4}$$

$$M'(0) = \frac{6}{4} = \frac{3}{2}, \quad M''(0) = \frac{24-12\cdot 2(-1)}{16} = 3$$

$\mu_1' = \frac{3}{2}$ and $\mu_2' = 3$

$$\sigma^2 = \mu_2' - (\mu_1')^2 = 3 - \frac{9}{4} = \frac{3}{4}$$

4.37 $\frac{1}{2}\int_{-\infty}^{0} e^{tx}e^{x}\,dx + \frac{1}{2}\int_0^\infty e^{tx}e^{-x}\,dx$ $\qquad y = -x$

$\qquad cy = -dx$

$$\frac{1}{2}\int_0^\infty e^{-ty}e^{-y}\,dy + \frac{1}{2}\int_0^\infty e^{tx}e^{-x}\,dx = \frac{1}{2}\int_0^\infty e^{-(1+t)y}\,dy + \frac{1}{2}\int_0^\infty e^{-(1-t)x}\,dx$$

$$= \frac{\frac{1}{2}\left[e^{-(1+t)y}\right]\Big|_0^\infty}{-(1+t)} + \frac{\frac{1}{2}\left[e^{-(1-t)y}\right]\Big|_0^\infty}{-(1-t)} = \frac{1}{2}\left[\frac{1}{1+t} + \frac{1}{1-t}\right]$$

$$= \frac{1/2}{(1+t)(1-t)}[1-t+1+t] = \frac{1}{1-t^2}$$