

MATH 421 – QUIZ 2 – FALL 2019 || 50 POINTS || 25 MINUTES

Instructions: Show all work. No collaboration or references.
No computational devices allowed without instructor permission.

Print Name _____

1. (6 pts) Consider the probability density function $f(x) = \alpha e^{-\alpha x}$ for $x > 0$, and let $t > 0$. Calculate $E(X | X > t)$.

2. (6 pts) You are given the joint probability density function $f(x, y) = \frac{1}{3}(4x + 3y^2)$ on $[0, 1]^2$. Find $E(X^2)$.

3. (6 pts) Consider the following joint probability mass function. Find the value of p so that $\text{Cov}(X, Y) = 0$.

		X	
		0	1
Y	0	0.3	$0.6 - p$
	1	0.1	p

4. (6 pts) You are given two exponentially distributed independent random variables $X_1 \sim \text{Exp}(\text{rate} = \alpha_1)$ and $X_2 \sim \text{Exp}(\text{rate} = \alpha_2)$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$. Find $\text{Cov}(Y_1, Y_2)$.

5. (6 pts) Consider random variable X which has moment generating function $M_X(t) = (1 - t\beta)^{-\alpha}$ for $0 \leq t < \frac{1}{\beta}$. Find $E(X)$ and $\text{Var}(X)$.

6. (10 pts) Consider an experiment which has probability of success 5% that is to be repeated over several trials.

(a) What is the probability that the first success occurs by the fourth trial? (*hint: Geometric.*)

(b) What is the probability that there are exactly 3 successes in ten trials? (*hint: Binomial.*)

7. (10 pts) You are given that insurance claims occur at random times at rate 20 per month and that the time elapsed between claims is modeled by an exponential random variable.

(a) What is the probability that there are 225 or fewer claims in a given year? (*hint: Poisson or Gamma.*)

(b) Suppose the insurance company has a fund with enough money to pay out 300 claims. How long is this fund expected to last? (*hint: Gamma.*)