4.40 $z=\frac{1}{4}(x-3)$, $a=-3$, $b=4$

$$M_z(t)=e^{-(3/4)t}\cdot e^{(3/4)t+(8/16)t^2}=e^{(1/2)t^2}$$

$$M_z(t)=1+\frac{1}{2}t^2+\ldots \qquad \mu=0 \text{ and } \sigma^2=1$$

4.44 $$E(x_2)=\int_0^1\int_0^1\int_0^\infty x_2(x_1+x_2)e^{-x_3}\,dx_3\,dx_2\,dx_1=\int_0^1\int_0^1(x_1^2+x_1x_2)dx_2\,dx_1$$

$$=\int_0^1\left(x_1^2x_2+x_1\frac{x_2^2}{2}\right|_0^1 dx_1=\int_0^1(x_1^2+\frac{1}{2}x_1)dx_1=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

$$E(x_3)=\int_0^1\int_0^1\int_0^\infty x_3e^{-x_3}(x_1+x_2)\,dx_3\,dx_1\,dx_2=\int_0^1\int_0^1(x_1+x_2)dx_1\,dx_2$$

$$=\int_0^1\left(\frac{1}{2}+x_2\right)dx_2=\frac{1}{2}+\frac{1}{2}=1$$

$$E(x_2x_3)=\int_0^\infty x_3e^{-x_3}dx_3\int_0^1\int_0^1(x_1^2+x_1x_2)dx_2\,dx_1$$

$$=\int_0^1(x_1^2+\frac{x_1}{2})dx_2=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

$$\text{cov}(x_1,x_3)=\frac{7}{12}-\frac{7}{12}\cdot 1=0$$

4.45 $$E(X)=\frac{1}{4}\int_0^1\int_0^2(2x^2+xy)\,dy\,dx=\frac{1}{4}\int_0^1(4x^2+2x)\,dx=\frac{1}{4}\left(\frac{4}{3}+1\right)=\frac{7}{12}$$

$$E(Y)=\frac{1}{4}\int_0^2\int_0^1(2xy+y^2)\,dx\,dy=\frac{1}{4}\int_0^2(y+y^2)\,dy=\frac{1}{4}\left(2+\frac{8}{3}\right)=\frac{14}{12}$$

$$E(XY)=\frac{1}{4}\int_0^1\int_0^2(2x^2y+xy^2)\,dy\,dx=\frac{1}{4}\int_0^1(4x^2+\frac{8}{3}x)\,dx=\frac{1}{4}\left(\frac{4}{3}+\frac{4}{3}\right)=\frac{2}{3}$$

$$\text{cov}(X,Y)=\frac{2}{3}-\frac{7}{12}\cdot\frac{14}{12}=\frac{2}{3}-\frac{49}{72}=-\frac{1}{72}$$

4.47 **(a)** $$E(U)=\int_{-1}^{0}(x+x^2)\,dx+\int_{0}^{1}(x-x^2)\,dx=-\frac{1}{2}+\frac{1}{3}+\frac{1}{2}-\frac{1}{3}=0$$

$$E(V)=\int_{-1}^{0}(x^2+x^3)\,dx+\int_{0}^{1}(x^2-x^3)\,dx=\frac{1}{3}-\frac{1}{4}+\frac{1}{3}-\frac{1}{4}=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$$

$$E(UV)=\int_{-1}^{0}(x^3+x^4)\,dx+\int_{0}^{1}(x^3-x^4)\,dx=-\frac{1}{4}+\frac{1}{5}+\frac{1}{4}-\frac{1}{5}=0$$

$$\text{cov}(U,V)=0-0\cdot\frac{1}{6}=0$$

not independent; in fact $V=U^2$.

4.49 **(a)** $$\mu_Y=2(4)-3(9)+4(3)=-7$$

$$\sigma_Y^2=4(3)+9(7)+16(5)=155$$

(b) $$\mu_Z=1(4)+2(9)-1(3)=19$$

$$\sigma_Z^2=1(3)+4(7)+1(5)=36$$

4.50 **(a)** $$\mu_Y=-7,\ \ \sigma_Y^2=155-12-48+48=143$$

(b) $$\mu_Z=19,\ \ \sigma_Z^2=36+4+6+8=54$$

4.51 $$E(x)=\frac{1}{3}\int_0^1\int_0^2(x^2+xy)\,dy\,dx=\frac{1}{3}\int_0^1(2x^2+2x)\,dx=\frac{1}{3}\left(\frac{2}{3}+1\right)=\frac{5}{9}$$

$$E(x^2)=\frac{1}{3}\int_0^1\int_0^2(x^3+x^2y)\,dy\,dx=\frac{1}{3}\int_0^1(2x^3+2x^2)\,dx=\frac{1}{3}\left(\frac{1}{2}+\frac{2}{3}\right)=\frac{7}{18}$$

$$\sigma_X^2=\frac{7}{18}-\frac{25}{81}=\frac{63-50}{162}=\frac{13}{162}$$

$$E(Y)=\frac{1}{3}\int_0^2\int_0^1(xy+y^2)\,dx\,dy=\frac{1}{3}\int_0^2\left(\frac{1}{2}y+y^2\right)dy=\frac{1}{3}\left(1+\frac{8}{3}\right)=\frac{11}{9}$$

$$E(Y^2)=\frac{1}{3}\int_0^2\int_0^1(xy^2+y^2)\,dx\,dy=\frac{1}{3}\int_0^2\left(\frac{1}{2}y^2+y^3\right)dy=\frac{1}{3}\left(\frac{4}{3}+4\right)=\frac{16}{9}$$

$$\sigma_Y^2=\frac{16}{9}-\frac{121}{81}=\frac{144-121}{81}=\frac{23}{81}$$

$$E(XY)=\frac{1}{3}\int_0^1\int_0^2(x^2y+xy^2)\,dy\,dx=\frac{1}{3}\int_0^1\left(2x^2+\frac{8}{3}x\right)dx=\frac{1}{3}\left(\frac{2}{3}+\frac{4}{3}\right)=\frac{2}{3}$$

$$\text{cov}(X,Y)=\frac{2}{3}-\frac{5}{9}\cdot\frac{11}{9}=-\frac{1}{81}$$

$$\text{var}(w)=9\cdot\frac{13}{162}+16\cdot\frac{23}{81}+24\cdot\left(-\frac{1}{81}\right)=\frac{177+736-48}{162}=\frac{805}{162}$$

4.56 $F(-1|-1) = \frac{1}{5},\ f(1|-1) = \frac{4}{5}$

$$\mu_{x|-1} = (-1)\frac{1}{5} + 1\cdot\frac{4}{5} = \frac{3}{5}$$

$$\mu_2' = 1 - \frac{1}{5} + 1\cdot\frac{4}{5} = 1 \qquad \sigma^2_{x|-1} = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$$

4.57 $f(z|1,2) = \phi(1|1,2) = \frac{1}{3},\ \phi(2|1,2) = \frac{2}{3}$

$$E(z^2|1,2) = 1^2\cdot\frac{1}{3} + 2^2\cdot\frac{2}{3} = \frac{1}{3} + \frac{8}{3} = 3$$

4.58 $f\left(y\middle|\ \frac{1}{4}\right) = \frac{1}{6}(2y+1) \qquad 0 < y < 2)$

$$\mu_{Y|1/4} = \frac{1}{6}\int_0^2 (2y^2 + y)\,dy = \frac{1}{6}\left(\frac{16}{3} + 2\right) = \frac{1}{6}\cdot\frac{22}{3} = \frac{11}{9}$$

$$\mu_2' = \frac{1}{6}\int_0^2 (2y^3 + y^2)\,dy = \frac{1}{6}\left(8 + \frac{8}{3}\right) = \frac{1}{6}\cdot\frac{32}{3} = \frac{16}{9}$$

$$\sigma^2_{Y|1/4} = \frac{16}{9} - \frac{121}{81} = \frac{23}{81}$$

4.60 (a) $f(x|a) \le x \le b) = \frac{f(x)}{F(b) - F(a)} \qquad a \le x < b)$

$$f(x|a \le x \le b) = \int_a^x \frac{f(x)}{F(b) - F(a)}\,dx = \frac{1}{F(b) - F(a)}\cdot F(x) - F(a)$$

$$= \frac{F(x) - F(a)}{F(b) - F(a)} \qquad a < x < b$$

(b) $f(x|a \le x \le b) = \frac{f(x)}{F(b) - F(a)}$

$$E[u(x)|a \le x \le b] = \frac{\int_a^b u(x) f(x)\,dx}{F(b) - F(a)} = \frac{\int_a^b u(x) f(x)\,dx}{\int_a^b f(x)\,dx}$$