5.22 $\sum_{x=1}^{\infty} \theta(1-\theta)^{x-1} = 1$

$\theta + \sum_{x=2}^{\infty} \theta(1-\theta)^{x-1} = 1 \qquad y = x-1$

$\sum_{y=1}^{\infty} \theta(1-\theta)^{y} = 1-\theta$

$\sum_{y=1}^{\infty} [(1-\theta)^{y} + \theta y(1-\theta)^{y-1}(-\theta)] - 1$

$\sum_{y=1}^{\infty} (1-\theta)^{y} - \sum_{y=1}^{\infty} \theta(1-\theta)^{y-1} = -1$

$\frac{1-\theta}{\theta} - \mu = -1$

$-\mu = -\frac{1}{\theta}$ and $\mu = \frac{1}{\theta}$

$\theta + \theta(1-\theta) + \sum_{x=3}^{\infty} \theta(1-\theta)^{x-1} = 1 \qquad y = x-2$

$\theta + \theta(1-\theta) + \sum_{y=1}^{\infty} \theta(1-\theta)^{y+1} = 1$

$\sum_{y=1}^{\infty} \theta(1-\theta)^{y+1} = 1-\theta-\theta(1-\theta) = (1-\theta)^2$

then differentiate *twice* with respect to θ.

5.23 $P(X = x+n | x > n) = \frac{P(X = x+n)}{P(X > n)} = \frac{\theta(1-\theta)^{x+n}}{(1-\theta)^n} = \theta(1-\theta)^x$ QED

$P(X > n) = \frac{\theta(1-\theta)^n}{1-(1-\theta)} = (1-\theta)^n$

5.24 $f(x) = \theta(1-\theta)^{x-1`} \quad F(x) = \sum_{t=1}^{x} \theta(1-\theta)^{tx-1} = \theta \cdot \frac{1-(1-\theta)^x}{1-(1-\theta)} = 1-(1-\theta)^x$

$z(x) = \frac{\theta(1-\theta)^{x-1}}{(1-\theta)^{x-1}} = \theta$

5.41 $\binom{5}{3}(0.1)^3(0.9)^2 + \binom{5}{4}(0.1)^4(0.9) + \binom{5}{5}(0.1)^5$

$= 10(0.001)(0.81) + 5(0.0001)(0.9) + (0.00001)$

$= 0.0081 + 0.00045 + 0.00001 = 0.0086$

5.50 **(a)** $\sigma_{\text{orig}} = \sqrt{np(1-p)}$. If $\sigma_{\text{new}} = \frac{1}{2}\sigma_{\text{orig}} = \frac{1}{2}\sqrt{np(1-p)} = \sqrt{\frac{n}{4}p(1-p)}$, then $n_{\text{new}} = \frac{1}{4}n_{\text{orig}}$

(b) $\sigma_{\text{orig}} = \sqrt{np(1-p)}$; $\sigma_{\text{new}} = \sqrt{nkp(1-p)} = \sqrt{k}\cdot\sqrt{np(1-p)} = \sqrt{k}\cdot\sigma_{\text{orig}}$

5.51 $P(x \geq 3) = 1 - b(0;20,0.05) - (b(1;20,0.05) - b(2;20,0.05)$

$= 1 - 0.3585 - 0.3774 - 0.1887 = 0.0754$

Thus, there is only a 0.0754 chance of obtaining 3 or more failures if the manufacturer's claim is correct.

5.60 $\theta = 0.05,\ x = 15,\ k = 2$

(a) $b^* = \binom{14}{1}(0.5)^2(0.95)^{13} = 14(0.0025)(0.51334) = 0.0180$

(b) $b^* = \frac{2}{15}\cdot(2;\ 15,\ 0.05) = \frac{2}{15}(0.1348) = 0.0180$

5.62 $g = (0.999)^{800}$

$\log g = 800(\log 0.999)$

$= 800(0.99957 - 1)$

$= 799.656 - 800 = 0.656 - 1$

$g = 0.4529$ (depends on rounding)

5.81 **(a)** $f(3;\ 5.2) = 0.1293$

(b) $0.0220 + 0.0104 + 0.0045 + 0.0018 + 0.0007 + 0.0002 + 0.0001 = 0.0397$

(c) $0.1681 + 0.1748 + 0.1515 = 0.4944$