6.1 $$\int_{\alpha}^{\alpha+p(\beta-\alpha)} \frac{1}{\beta-a}\,dx = \frac{1}{\beta-\alpha}[\alpha+p(\beta-\alpha)-\alpha] = p$$

6.2 $$\mu = \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}\frac{1}{\beta-a}\,x\,dx = \frac{1}{\beta-\alpha}\left(\frac{\beta^2}{2}-\frac{\alpha^2}{2}\right) = \frac{1}{2(\beta-\alpha)}\cdot(\beta-\alpha)(\beta+\alpha) = \frac{\alpha+\beta}{2}$$

$$\mu_2' = \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} x^2\,dx = \frac{1}{3(\beta-\alpha)}(\beta^3-\alpha^3) = \frac{1}{3}(\beta^2+\alpha\beta+\alpha^2)$$

$$\sigma^2 = \frac{1}{3}(\beta^2+\alpha\beta+\alpha^2) - \frac{(\alpha+\beta)^2}{4} = \frac{1}{12}[4\beta^2+4\alpha\beta+4\alpha^2-3\alpha^2-6\alpha\beta-3\beta^2)$$
$$= \frac{1}{12}(\beta^2-2\alpha\beta+\alpha^2) = \frac{1}{12}(\beta-\alpha)^2$$

6.3 $$F(x) = \frac{1}{\beta-\alpha}\int_{\alpha}^{x} dx = \frac{x-\alpha}{\beta-\alpha} \qquad f(x) = \begin{cases} 0 & x \le \alpha \\ \dfrac{x-\alpha}{\beta-\alpha} & \alpha < x < \beta \\ 1 & \beta \le x \end{cases}$$

6.15 $$f(x) = \frac{1}{\theta}e^{-x/\theta} \qquad p = \int_{0}^{-\theta\ln(1-p)} \frac{1}{\theta}e^{-x/\theta}d\theta = [-e^{-x/\theta}]\Big|_{0}^{-\theta\ln(1-p)}$$
$$= 1-e^{\ln(1-p)} = 1-(1-p) = p$$

6.16 $$\frac{p(x\ge t+T)}{P(x\ge T)} = \frac{e^{-(t+T)/\theta}}{e^{-T/\theta}} = e^{-t/\theta} = p(x\ge t)$$

6.17 $$M_x = (1-\theta t)^{-1} \qquad M_{x-\theta} = e^{-\theta t}(1-\theta t)^{-1} = \frac{e^{-\theta t}}{1-\theta t}$$

6.18 $$\left(1-\theta t+\frac{\theta^2t^2}{2!}-\frac{\theta^3t^3}{3!}+\frac{\theta^4t^4}{4!}\ldots\right)(1+\theta t+\theta^2t^2+\theta^3t^3+\theta^4t^4\ldots)$$
$$1+\left(1+\frac{1}{2}-1\right)\theta^2t^2+\left(-\frac{1}{6}+\frac{1}{2}-1+1\right)\theta^3t^3+\left(\frac{1}{24}-\frac{1}{6}+\frac{1}{2}-1+1\right)\theta^4t^4\ldots$$

$$1+\frac{\theta^2t^2}{2!}+2\cdot\frac{\theta^3t^3}{3!}+\frac{9\theta^4t^4}{4!}+\ldots \qquad \alpha_3 = \frac{2\theta^3}{\theta^3} = 2 \qquad \alpha_4 = \frac{9\theta^4}{\theta^4} = 9$$

6.21 $\mu'_r = \alpha \int_1 x^{r-\alpha-1}\, dx$ exists only if $r-\alpha-1<1$

$r<\alpha-2$

6.22 $\mu'_1 = \alpha \int_1^{\infty} x^{-\alpha}\, dx = \alpha \frac{x^{1-\alpha}}{1-\alpha}\Big|_1^{\infty} = \frac{\alpha}{\alpha-1}$

6.23 **(a)** $k\int_0^{\infty} x^{\beta-1}e^{-\alpha x^{\beta}}\, dx = 1$ let $u = \alpha x^{\beta}$ $du = \alpha\beta x^{\beta-1}\, dx$

$$= k\int_0^{\infty} \frac{1}{\alpha\beta} e^{-u}\, du = \frac{k}{\alpha\beta} = 1 \qquad k = \alpha\beta$$

(b) $\mu = \alpha\beta \int_0^{\infty} x^{\beta}\, e^{-\alpha x^{\beta}}\, dx$

$$= \alpha^{-1/\beta}\int u^{1/\beta}\, e^{-u}\, du = \alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right)$$

6.24 **(a)** $f(x) = \frac{1}{\theta}e^{-x/\theta}$ $F(t) = \int_0^{t/\theta} e^{-u}\, du = 1-e^{-t/\theta} = 1-e^{-t/\theta}$

$$\frac{f(t)}{1-F(t)} = \frac{\frac{1}{\theta}e^{-t/\theta}}{e^{-t/\theta}} = \frac{1}{\theta}$$

(b) $F(t) = \alpha\beta\int_0^{t} x^{\beta-1}\, e^{-\alpha x^{\beta}}\, dx = 1-e^{-\alpha t^{\beta}}$

$$\frac{f(t)}{1-F(t)} = \frac{\alpha\beta t^{\beta-1}\, e^{-\alpha t^{\beta}}}{e^{-\alpha t^{\beta}}} = \alpha\beta t^{\beta-1}$$

6.35 $\alpha_3 = 0$ and $\alpha_4 = \dfrac{3\sigma^4}{\sigma^4} = 3$

6.36 $M_x(t) = e^{\mu t + (1/2)\sigma^2 t^2}$

$M_{(x-\mu)/\sigma} = e^{-(\mu/\sigma)t} \cdot e^{\mu(t/\sigma) + (1/2)\sigma^2 (t/\sigma)^2} = e^{(1/2)t^2}$

6.37 $E(x) = \mu,\ E(x^2) = \sigma^2 + \mu^2,\ E(x^3) = \mu^3 + 3\mu\sigma^2$

$\text{cov}(x, x^2) = (\mu^3 + 3\mu\sigma^2) - \mu(\sigma^2 + \mu^2) = 2\mu\sigma^2$

for standard normal distribution $\mu = 0 \rightarrow \text{cov}(x, x^2) = 0$

6.38 $M = e^{(1/2)t^2} = 1 + \dfrac{\left(\frac{1}{2}t^2\right)}{1!} + \dfrac{\left(\frac{1}{2}t^2\right)^2}{2!} + \ldots \dfrac{\left(\frac{1}{2}t^2\right)^{r/2}}{(r/2)!}$

$\downarrow$

$\dfrac{t^r}{2^{r/2}(r/2)!} = \dfrac{r!}{2^{r/2}(r/2)!} \cdot \dfrac{t^r}{r!}$

(a) $\mu_r = 0$ since coefficient of t with r odd is zero.

(b) $\mu_r = \dfrac{r!}{(r/2)!} \dfrac{1}{2^{r/2}}$ read off for r even.

6.41 $M_x(t) = e^{\lambda(e^t - 1)} \qquad \mu = \lambda,\ \sigma = \sqrt{\lambda}$

$M_{(x-\mu)/\sigma}(t) = e^{-(\mu/\sigma)t} M_x\left(\dfrac{t}{\sigma}\right) = e^{-\sqrt{\lambda}t} e^{\lambda(e^{t/\sigma} - 1)}$

$\ln M_{(x-\mu)/\sigma}(t) = -\sqrt{\lambda}t + \lambda(e^{t/\sigma} - 1)$

$= -\sqrt{\lambda}t + \lambda(e^{t/\sqrt{x}} - 1)$

$= -\sqrt{\lambda}t + \lambda\left[\dfrac{t}{\sqrt{\lambda}} + \dfrac{t^2}{2\lambda} + \dfrac{t^3}{3\lambda\sqrt{\lambda}} + \ldots\right]$

$= -\sqrt{\lambda}t + \sqrt{\lambda}t + \dfrac{t^2}{2} + \dfrac{t^3}{3\sqrt{\lambda}} + \ldots$

$\lambda \rightarrow \infty \qquad = \dfrac{1}{2}t^2$

6.42 $M_x(t) = (1-\beta t)^{-\alpha}$ $\qquad \mu = \alpha\beta,\ \sigma = \beta\sqrt{\alpha}$

$$M_{(x-\mu)/\sigma} = e^{-\sqrt{\alpha}t}\left(1-\frac{t}{\sqrt{\alpha}}\right)^{-\alpha}$$

$$\ln M_{(x-\mu)/\alpha} = -\sqrt{\alpha}t - \alpha\ln\left(1-\frac{t}{\sqrt{\alpha}}\right) \qquad \ln(1+z) = +z+\frac{z^2}{2}+\frac{z^3}{3}+\ldots$$

$$= -\sqrt{\alpha}t + \alpha\left[\frac{t}{\sqrt{a}} - \frac{t^2}{2\alpha} + \frac{t^3}{3\alpha\sqrt{\alpha}}\ldots\right] = +\frac{t^2}{2} \text{ when } \alpha \to \infty$$

6.47 $\mu_1 = 2,\ \mu_2 = 5,\ \sigma_1 = 3,\ \sigma_2 = 6,\ p = \frac{2}{3}$

$$\mu_{Y|1} = 5 + \frac{2}{3}\cdot\frac{6}{3}(1-2) = 5 - \frac{4}{3} = \frac{11}{3}$$

$$\sigma^2_{Y|1} = 36\left(1-\frac{4}{9}\right) = \frac{36\cdot 5}{9} = 20 \qquad \sigma_{Y|1} = \sqrt{20} = 4.47$$

6.54 **(a)** $\displaystyle\int_0^{24}\frac{1}{120}e^{-(1/120)x}dx = -e^{-x/120}\Big|_0^{24} = 1-e^{-0.2} = 1-0.8187 = 0.1813$

(b) $\displaystyle\int_{180}^{\infty}\frac{1}{120}e^{-1/120}dx = -e^{-x/120}\Big|_{180}^{\infty} = e^{-1.5} = 0.2231$

6.59 $\lambda = 0.5 \displaystyle\int_3^{\infty} e^{-0.5t}dt = -e^{-0.5t}\Big|_3^{B} = e^{-1.5} = 0.2231$

6.61 $\alpha = 0.025,\ \beta = 0.5$

(a) $\mu = (0.025)^{-2}\Gamma(3) = \dfrac{2}{(0.025)^2} = 3200$ hours

(b) $\alpha\beta\displaystyle\int_{4000}^{\infty} x^{\beta-1}e^{-\alpha x^{\beta}}dx$ $\qquad y = \alpha x^{\beta} \qquad y = 0.025\cdot\sqrt{4000} = 1.58$

$dy = \alpha\beta x^{\beta-1}dx$

$$= \int_{1.58}^{\infty} e^{-y}dy = e^{-1.58} = 0.2060$$

6.63 **(a)** $0.5 - 0.3729 = 0.1271$

(b) $0.5 + 0.1406 = 0.6406$

(c) $0.1772 - 0.359 = 0.1413$

(d) $0.2190 + 0.3686 = 0.5876$

6.67 **(a)** $z_{0.05} = 1.645$ 0.4500

(b) $z_{0.025} = 1.96$ 0.475

(c) $z_{0.01} = 2.33$ 0.49

(d) $z_{0.005} = 2.575$ 0.495

6.80 **(a)** $\mu = 50,\ \sigma = 5,\ z = \dfrac{51.5 - 50}{5} = 0.3$

49 to 51 $2(0.1179) = 0.2358 = 0.24$

(b) $\mu = 500,\ \sigma = 15.81,\ z = \dfrac{510.5 - 500}{15.81} = 0.664$

490 to 510 $2(0.2454) = 0.49$

(c) $\mu = 5000,\ \sigma = 50,\ z = \dfrac{5100.5 - 5000}{50} = 2.01$

4900 to 5100 $2(0.4778) = 0.96$