BONUS (10 bonus pts max) Choose one of the following three bonus problems to attempt.

I. Prove that the moment generating function $M_X(t)$ is concave up for all t.

II. You are given moment generating function $M_X(t) = \frac{1}{2} + \frac{1}{4}e^t + \frac{1}{8}e^{2t} + \frac{1}{8}e^{3t}$. Find the probability function for X.

III. Let $X \sim \text{Pois}(\lambda)$ and $Y \sim \text{Gamma}(k, \frac{t}{\lambda})$. Prove that $\text{P}(X < k) = \text{P}(Y \geq t)$ for any positive integer k, and $t > 0$.