# Genome Inversions and Graph Pressing Sequences

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## Introduction

An inversion is a chromosome rearrangement in which a segment of a chromosome is reversed end to end. An inversion occurs when a single chromosome undergoes breakage and rearrangement within itself.

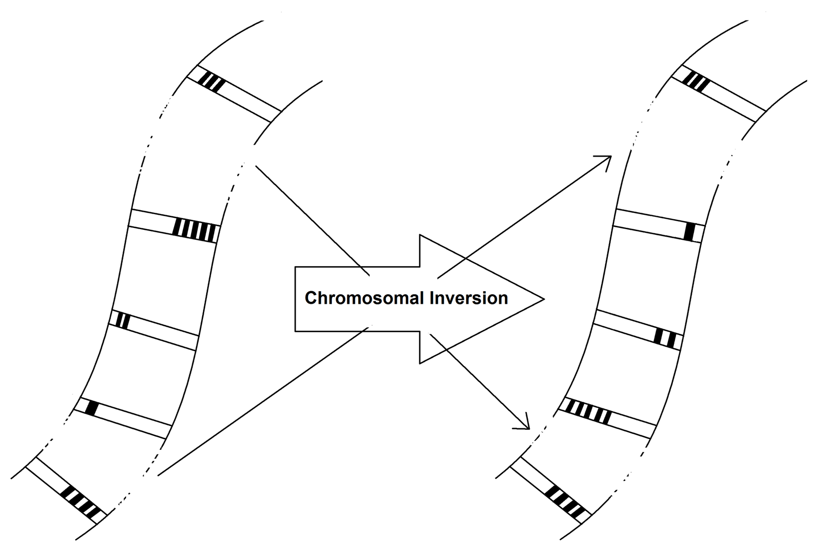



Figure 1: A chromosomal inversion

One can construct a useful metric on (homologous) genome sequences by computing the minimal number inversions needed to obtain a genome sequence from another.

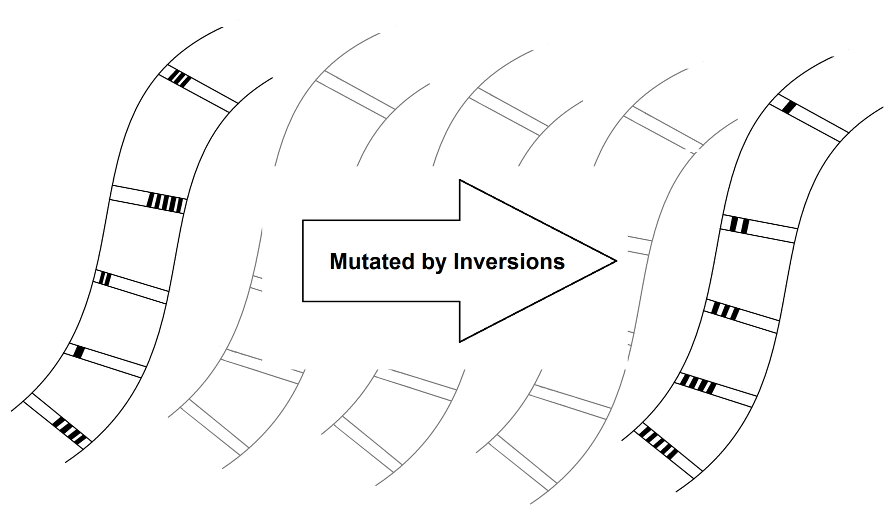



Figure 2: Iterated Chromosomal Rearrangements

In mathematics, we interpret the sequences as signed permutations (signed since DNA is oriented) and encode the inversions as permutation reversals.

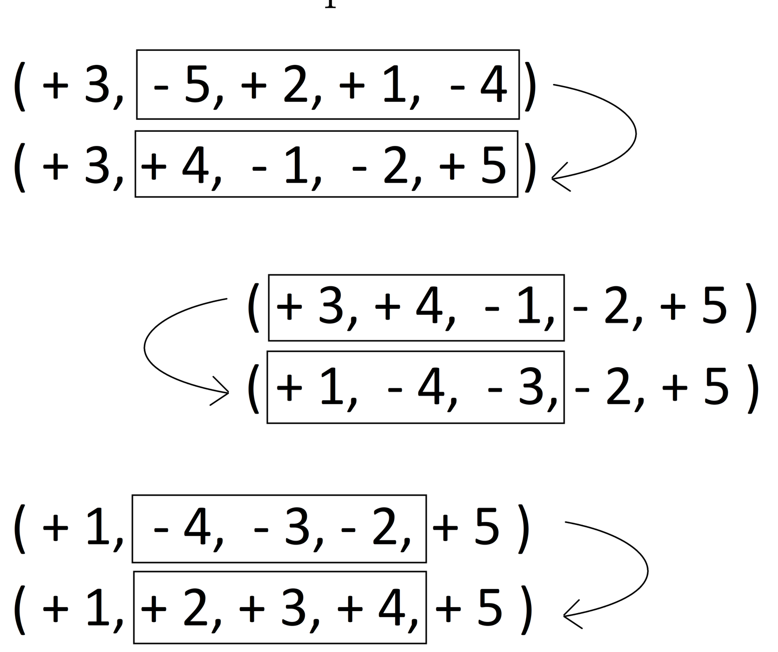
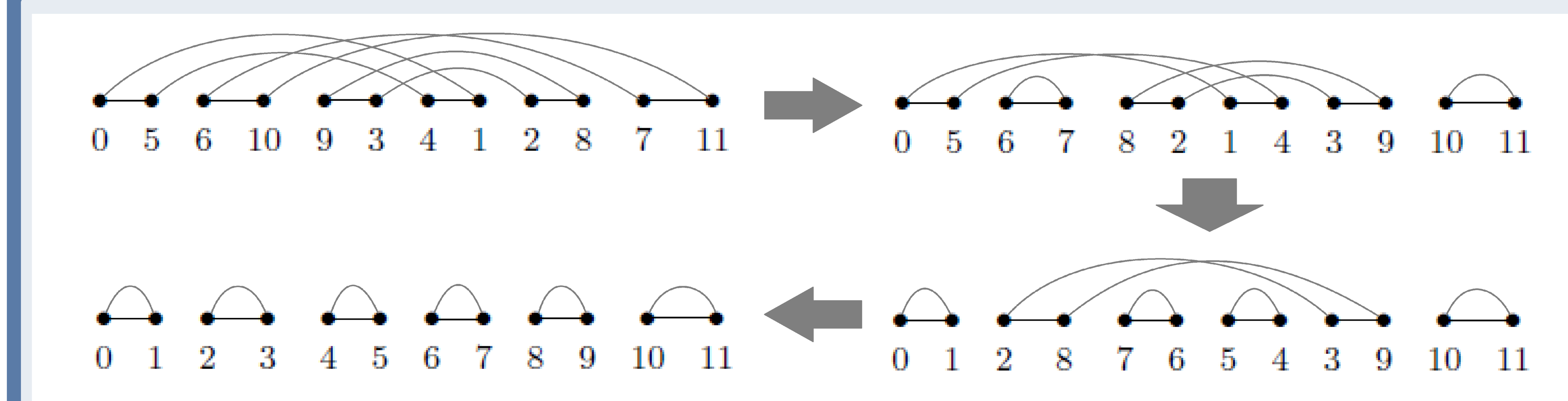



Figure 3: One possible explanation for Figure 2

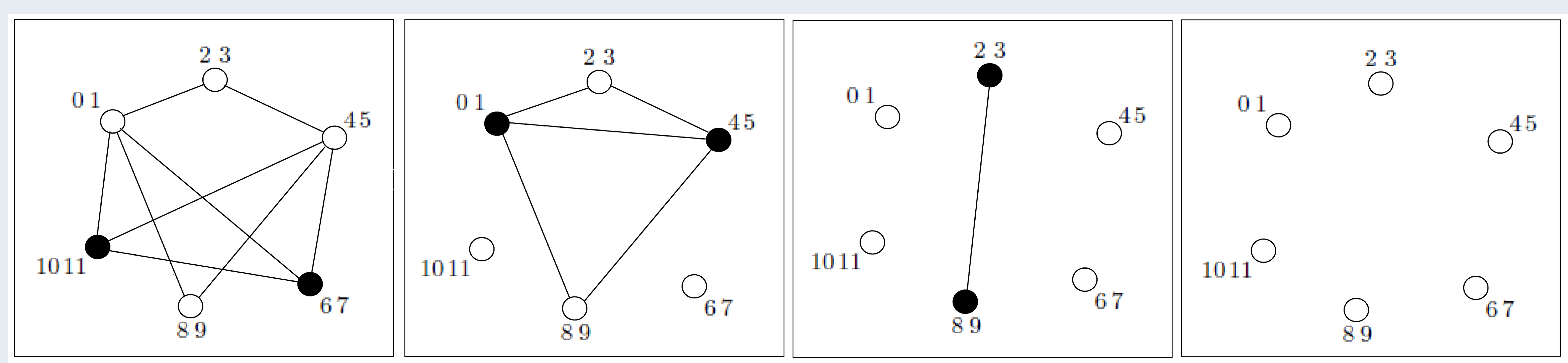
From the signed permutation we draw a graph of "reality and desire". To obtain this drawing we replace each integer $k$ with the pair $\{2k, 2k+1\}$ and write them in increasing order exactly when $k$ carried the sign $+$. We then augment the resulting permutation on the left by appending $0$ and on the right by appending $2n+1$. The resulting non-negative permutation $\tau = (\tau_0, \tau_1, \ldots, \tau_{2n+1})$ is used to draw a graph with edge set $\{\{\tau_{2k}, \tau_{2k+1}\} \cup \{2k, 2k+1\}\} \mid k \in [0, n]\}$.

## Graphs of Reality and Desire




From the drawing of reality and desire we obtain a bicolored graph with vertex set $\{v = \{2k, 2k+1\} \mid k \in [0, n]\}$ and edge set $E$ where $uv$ is an edge whenever the edges corresponding to $u$ and $v$ cross in the drawing of the graph of reality and desire.

## Bicolored Graphs




In our research we have focused on studying the successful pressing sequences of the bicolored graphs, as well as their adjacency matrices. In particular, we are interested in determining how many successful pressing sequences exists for a given graph.

## Main Results

1) Let $\sigma$ be a permutation, $A$ the adjacency matrix of a bicolored graph $G$, and $P$ the permutation matrix encoding $\sigma$. Then $\sigma$ is a successful pressing sequence of $G$ exactly when $P^T AP$ was a unique instructional Cholesky factorization over $\mathbb{F}_2$.
2) Let $G$ be a bicolored graph. There exists a set of posets (generated by $G$) whose linear extensions partition the set of successful pressing sequences of $G$.
3) The number of pressing sequences of a bicolored graph can be determined in exponential time.
4) Up to isomorphism, there are $(3-(-1)^n)/2 \cdot 3^{\lfloor n/2 \rfloor - 1}$ uniquely pressable connected graphs on $n$ vertices.

## Other Results/Future Directions

1) Determining if a graph is "uniquely pressable" can be determined in cubic time.
2) Some bicolored graphs generate only one poset (whose linear extensions are the successful pressing sequences of the graph). These posets are exactly the induced $N$-free and bowtie-free posets.

It is desirable to determine the complexity of counting the successful pressing sequences of a bicolored graph. So far we have shown that the complexity is no more than exponential (as opposed to the complexity of counting linear extensions of a poset which is super-exponential). Since the posets that can be generated by themselves are a subset of the series-parallel posets, a direct reduction from linear extensions is unlikely. If the complexity is shown to be $\#-P$ hard (or while it remains undetermined) it is desirable to find an FPRAS for estimating the number of successful pressing sequences of a bicolored graph.

## References

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## Contact Information

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Give it a try: http://pressing-game.info