

### Misc. formulas:

$$nP_k = \frac{n!}{(n-k)!} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \binom{n}{n_1, n_2, \dots, n_k} = \frac{n!}{n_1! n_2! \dots n_k!}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i) - \sum_{i < j} P(A_i \cap A_j) + \sum_{i < j < k} P(A_i \cap A_j \cap A_k) + \dots + (-1)^{n-1} \sum_{i < \dots < n} P\left(\bigcap_{i=1}^n A_i\right)$$

$$P(B | A) = \frac{P(A | B)P(B)}{P(A | B)P(B) + P(A | B^c)P(B^c)} \quad P(B_k | A) = \frac{P(A | B_k)P(B_k)}{\sum_{i=1}^n P(A | B_i)P(B_i)}$$

$$\text{Var}(X) = E[(X - E(X))^2] = E(X^2) - [E(X)]^2 = E(X^2) - \mu_X^2 \quad \text{Var}(X) = \text{Cov}(X, X)$$

$$\text{Var}(u(X, Y)) = E[(u(X, Y) - E(u(X, Y)))^2] = E(u(X, Y)^2) - [E(u(X, Y))]^2$$

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] \quad \text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$$

$$M_X(t) = E[e^{tX}] \quad m_n = \mu'_n = E(X^n) \quad \mu_n = E[(X - \mu_X)^n] \quad M_{\frac{X+a}{b}} = e^{\frac{at}{b}} M_X\left(\frac{t}{b}\right)$$

If  $Y_1 = \sum_{i=1}^n a_i X_i$  and  $Y_2 = \sum_{i=1}^n b_i X_i$  then:

$$E(Y_1) = \sum_{i=1}^n a_i E(X_i)$$

$$\text{Var}(Y_1) = \sum_{i=1}^n a_i^2 \text{Var}(X_i) + 2 \sum_{i < j} a_i b_j \text{Cov}(X_i, X_j)$$

$$\text{Cov}(Y_1, Y_2) = \sum_{i=1}^n a_i b_i \text{Var}(X_i) + \sum_{i < j} (a_i b_j + a_j b_i) \text{Cov}(X_i, X_j)$$

$$E(X | a < X < b) = \frac{\int_a^b x f_X(x) dx}{\int_a^b f_X(x) dx} \quad E(X | c < Y < d) = \frac{\int_{-\infty}^{\infty} \int_c^d x f(x, y) dy dx}{\int_{-\infty}^{\infty} \int_c^d f(x, y) dy dx}$$

$$E(X | Y = y) = \int_{-\infty}^{\infty} x f_{X|Y=y}(x|y) dx = \int_{-\infty}^{\infty} x \frac{f(x, y)}{f_Y(y)} dx$$

$$E(u(X, Y) | a < X < b, c < Y < d) = \frac{\int_c^d \int_a^b u(x, y) f(x, y) dx dy}{\int_c^d \int_a^b f(x, y) dx dy}$$

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt \quad \Gamma(n) = (n-1)! \quad (2n-1)!! = \frac{(2n)!}{2^n n!} \quad (2n)!! = 2^n n!$$

$$Y = u(X) \quad f_Y(y) = f_X(u^{-1}(y)) \left| \frac{d}{dy} u^{-1}(y) \right|$$

### Distributions:

$$f_X(x) = \binom{n}{x} \theta^x (1-\theta)^{n-x} \text{ for } x = 0, 1, 2, \dots, n \quad E(X) = n\theta \quad \text{Var}(X) = n\theta(1-\theta) \quad M_X(t) = (1-\theta + \theta e^t)^n$$

$$f_X(x) = \theta(1-\theta)^{x-1} \text{ for } x = 1, 2, 3, \dots \quad E(X) = \frac{1}{\theta} \quad \text{Var}(X) = \frac{(1-\theta)}{\theta^2} \quad M_X(t) = \frac{\theta e^t}{1-\theta+\theta e^t}$$

$$f_X(x) = \binom{x-1}{k-1} \theta^k (1-\theta)^{x-k} \text{ for } x = k, k+1, k+2, \dots \quad E(X) = \frac{k}{\theta} \quad \text{Var}(X) = \frac{k(1-\theta)}{\theta^2} \quad M_X(t) = \left( \frac{\theta e^t}{1-\theta+\theta e^t} \right)^k$$

$$f_X(x) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ for } x = 0, 1, 2, \dots \quad E(X) = \lambda \quad \text{Var}(X) = \lambda \quad M_X(t) = e^{\lambda(e^t-1)}$$

$$f_X(x) = \frac{1}{\beta-\alpha} \text{ for } \alpha < x < \beta \quad E(X) = \frac{1}{2}(\alpha + \beta) \quad \text{Var}(X) = \frac{1}{12}(\beta - \alpha)^2 \quad M_X(t) = \frac{1}{\beta-\alpha} \frac{1}{t} (e^{t\beta} - e^{t\alpha})$$

$$f_X(x) = \alpha e^{-\alpha x} \text{ or } f_X(x) = \frac{1}{\theta} e^{-x/\theta} \text{ for } x \geq 0 \quad E(X) = \frac{1}{\alpha} = \theta \quad \text{Var}(X) = \frac{1}{\alpha^2} = \theta^2 \quad M_X(t) = (1 - \frac{1}{\alpha} t)^{-1} = (1 - \theta t)^{-1}$$

$$f_X(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \text{ for } x \geq 0 \quad E(X) = \alpha\beta \quad \text{Var}(X) = \alpha\beta^2 \quad M_X(t) = (1 - \beta t)^{-\alpha}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \text{ for } -\infty < x < \infty \quad E(X) = \mu \quad \text{Var}(X) = \sigma^2 \quad M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2} \quad \mu_{2n} = (2n-1)!! \sigma^{2n}$$

$$f_X(x) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2} \text{ for } x \geq 0 \quad E(X) = \nu \quad \text{Var}(X) = 2\nu \quad M_X(t) = (1 - 2t)^{-\nu/2}$$