MATH 421 – FALL 2019 – FINAL EXAM p-I (take home)|| 100 POINTS || 50 MINUTES

INSTRUCTIONS: You must show complete, correct work and reasoning in order to receive full credit. Write legibly, and clearly mark your final answers. Textbook and course notes are allowed as a reference. Do not seek assistance from any person or any other resource. Present only your own work.

Print Name: ______________________________

Signature: ______________________________

1. (8 pts) You have 3 indistinguishable balls to be placed into 3 distiguishable bins. How many distinguishable outcomes are there to do this? Assume each bin can hold any number of balls. List/sketch all outcomes. *(Hint: Draw indistinguishable balls as ◯ ◯ ◯ and distinguishable bins as* $\underset{1}{\sqcup}\underset{2}{\sqcup}\underset{3}{\sqcup}$*.)*

2. (8 pts) A group of 5 people are to select drinks from a cafe menu containing 10 distinct options.
   (a) Assume cafe supply is sufficient so that any person can get any drink. How many distinguishable outcomes are possible?

   (b) If instead, there is only one of each type of drink available, then how many distinguishable outcomes are there?

3. (8 pts) Consider an urn that contains 5 green, 4 blue, and 3 red balls. Two balls are drawn sequentially without replacement. Given that the 2^{nd} ball is red, what is the probability that the 1^{st} ball is green?

4. (24 pts) We are to consider worldwide earthquakes above Richter scale magnitude 5. We'll refer to these as M5+ earthquakes. Assume that the number of such earthquakes in any given 24 hour period is modeled by a Poisson random variable with mean 4.

   (a) Calculate the probability that there are at least 3 M5+ earthquake in a two day period.

   (b) What is the probability that the next M5+ earthquake occurs within the next hour?

   (c) What is the probability of the next 10 M5+ earthquakes occurring in a period of less than 24 hours?

5. (8 pts) Consider probability density function $f_X(x) = \frac{3}{8}x^2$ on $[0, 2]$. Find the pdf for $Y = \sqrt{\frac{X}{2}}$.

6. (8 pts) Assume that it is known that the contaminant content of a water supply is approximately normally distributed with mean 30 ppb (parts per billion) and standard deviation 4 ppb. If 64 samples are taken, find the approximate probability that the sample mean is greater than 31 ppb.

7. (8 pts) You are given moment generating functions $M_X(t) = e^{2t+t^2}$ and $M_Y(t) = \frac{1}{1-t}$ for jointly distributed random variables X and Y and that $\mathrm{Cov}(X, Y) = 1$. Find $\mathrm{E}(XY)$.

8. (10 pts) Consider factory production of widgets where there is a 0.1% chance of each widget being defective. What is the probability that at least 1 thousand widgets are produced before the first defective one?

9. (18 pts) Consider joint probability density function $f_{X,Y}(x, y) = \frac{4}{5}(x + y + xy)$ on $[0, 1]^2$.

   (a) Find the marginal pdf for X.

   (b) Find the conditional pdf for Y given $X = \frac{1}{3}$.

   (c) Calculate the conditional expected value of Y given $X = \frac{1}{3}$.