MATH 422 – QUIZ 1 – SPRING 2020 || 50 POINTS || 25 MINUTES

Instructions: Show all work. No collaboration. Textbook and course notes allowed as references. Computational devices allowed.	Print Name ______________________________

1. (10 pts) Let $X_1, ..., X_{10}$ be independent and identically normally distributed with mean μ and variance σ^2. Calculate the probability that the sample variance is more than twice the population variance. Write down an integral that can be used to calculate this (you do not need to evaluate it by hand). Be sure to write down any R code you use, or tell me how you evaluated this if you used other software.

2. (10 pts) Consider the distribution $f_X(x) = \theta x^{\theta-1}$ on $0 < x < 1$. Determine what values for parameter θ are allowable. Find both the *method of moments* and *maximum likelihood* estimators for θ. Apply both estimators to the dataset: $\{0.70, 0.90, 0.99, 0.60, 0.62, 0.31, 0.67, 0.71, 0.69, 0.95\}$.

3. (10 pts) Show that $\hat{\mu} = \frac{1}{n+1} \sum_{i=1}^{n} X_i$ is a consistent estimator of $\mu = \mathrm{E}(X)$. *(Hint: Just show that its mean square error goes to zero as the sample size goes to infinity.)*

4. (10 pts) Let $X_{(k)}$ be the k^{th} order statistic from a sample of size n from the distribution from question 2. Write down the formula for the probability density function of $X_{(k)}$. Find the probability that the minimum is less than 0.01 when $\theta = 2$ and $n = 3$, $\mathrm{P}(X_{(1)} < 0.01)$.

5. (10 pts) Consider the dataset below sampled form a normally distributed population.

97.6	99.6	96.9	100.7	100	110.7	99.1	94.7	92.4	98.8

(a) Estimate μ and σ^2 using the minimum variance unbiased estimators.

(b) Construct 95% confidence intervals for both μ and σ^2.