Comparing two populations:

$X_i \sim N(\mu_1, \sigma_1^2)$ size n

$Y_i \sim N(\mu_2, \sigma_2^2)$ size m

want to know if $\mu_1 = \mu_2$

$$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}\right)$$

Thus if we knew σ_1^2 & σ_2^2, then

$$Z = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} \sim N(0,1)$$

but we typically don't know pop. variances.
If n AND m are both large (n & $m \geq 30$)
then we can assume $\sigma_1^2 \approx s_1^2$ and $\sigma_2^2 \approx s_2^2$

and use $$Z \approx \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}} \overset{\text{approx}}{\sim} N(0,1)$$

In truth: the distr. of

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}}$$ is unknown, it's not t-distr. !!

But we have the Welch-Satterthwaite approx:

$$\frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}}} \overset{\text{approx}}{\sim} T_\nu$$

w/ $$\nu = \frac{\left(\frac{S_1^2}{n_0}+\frac{S_2^2}{m}\right)^2}{\left(\frac{S_1^2}{n}\right)^2\cdot\frac{1}{n-1}+\left(\frac{S_2^2}{m}\right)^2\cdot\frac{1}{m-1}}$$ (round down reccomended)

So: Hypothesis test:

$H_0: \mu_1=\mu_2$

$H_1: \mu_1\neq\mu_2$

w/ test statistic:

$$t=\frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_1^2}{n}+\frac{s_2^2}{m}}}$$

note: $\mu_1-\mu_2=0$ is H_0.

p-value $= 2\cdot pt(-|t|, \nu)$

In R if we have datasets: x & y

this is done w/ `t.test(x, y)`

@ $\alpha=0.05$.

Can adjust for $H_1: \mu_1>\mu_2$ or $H_1: \mu_1<\mu_2$ also

`t.test(x, y, mu=`Δ_0`, alternative="greater", conf.level=`$1-\alpha$`)`

paired t-test

If X & Y are paired, e.g. before & after or same object measured at diff. times or under diff. conditions.

e.g. patient before treatment & after treatment

$H_0: \mu_1 \le \mu_2$
$H_1: \mu_1 > \mu_2$

X = before, Y = after → hyp. is that treatment increases mean observation.

test statistic:

$$t = \frac{\bar{D} - 0}{\frac{s_D}{\sqrt{n}}}$$

w/ $D_1 = Y_1 - X_1$, $D_2 = Y_2 - X_2$, ..., $D_n = Y_n - X_n$

s_D = std. dev. of the D_i

& do regular 1-mean t-test.

$H_0: D > 0$
$H_1: D \le 0$

2-sample prop-test:

$$X \sim Bin(n, p_1)$$
$$Y \sim Bin(m, p_2)$$

$$\frac{X}{n} - \frac{Y}{m} \overset{approx}{\sim} N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n} + \frac{p_2(1-p_2)}{m}\right)$$

Rule of thumb:

$n \geq 30$, $m \geq 30$

$p_1 n$ & $(1-p_1)n \geq 5$

$p_2 m$ & $(1-p_2)m \geq 5$

Hypothesis test:

$H_0: p_1 = p_2$
$H_1: p_1 \neq p_2$

test stat:

$$z = \frac{\frac{X}{n} - \frac{Y}{m}}{\sqrt{\hat{p}(1-\hat{p}) \cdot \left(\frac{1}{n} + \frac{1}{m}\right)}}$$

w/ pooled proportion: $\hat{p} = \frac{X+Y}{n+m}$

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for $H_0: p_1 \geq p_2 + \Delta_0$
$H_1: p_1 < p_2 + \Delta_0$

use:

$$z = \frac{\frac{X}{n} - \frac{Y}{m} - \Delta_0}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_0} + \frac{\hat{p}_2(1-\hat{p}_2)}{m}}}$$

$\hat{p}_1 = \frac{X}{n}$, $\hat{p}_2 = \frac{Y}{m}$

ex: Biden support in Florida vs. Pennsylvania

Fox News poll

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

FL: $n = 1004$, $p_1 = 0.46$

PA: $m = 803$, $p_2 = 0.50$

w/ p_1 & p_2 close & n, m close & large

pooled vs non-pooled are similar

$$\hat{p} = \frac{(0.46)(1004) + (0.50)(803)}{1004 + 803} \approx 0.478$$

$$z = \frac{0.46 - 0.50}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n} + \frac{1}{m}\right)}}$$

$$\approx -1.69$$

pval = 2*pnorm(-1.69) ≈ 0.091 $\Rightarrow$ can't reject $H_0: p_1 = p_2$

ex 2 batches of medicine

$n = 7$ w/ $\overline{X} = 101.3$, $s_1 = 2.8$

$m = 10$ w/ $\overline{Y} = 99.8$, $s_2 = 3.1$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$t = \frac{101.3 - 99.8}{\sqrt{\frac{(2.8)^2}{7} + \frac{(3.1)^2}{10}}} \approx 1.04$$

$\nu = 13.9 \Rightarrow$ use $\boxed{\nu = 13}$

p-val $= 2 \cdot pt(-1.04, 13)$

$= 0.31 \Rightarrow$ can't reject H_0

Batches are very similar.