MATH 413 – QUIZ 1 – Fall 2020 || 50 POINTS

Instructions:

- Due Wednesday 10/7/2020 by 5pm.
- Either turn in your quiz physically, or as a digital pdf file on blackboard. Name the digital file precisely as follows: "`quiz01_math413_lastname.pdf`" with "`lastname`" replaced by your surname (i.e. family name, last name).
- You may use your course notes, my notes, and our course textbook as references.
- No collaboration allowed.
- No computational devices allowed.
- When asked to prove convergence or divergence, construct direct proofs using the appropriate definition of convergence or divergence, i.e. do not use not Remark 2.1.8, Theorem 2.2.1, Corollary 2.2.4, or any other results that similarly simplify the argument significantly.

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1. (5 pts) Consider the number system $\widetilde{\mathbb{R}} = \mathbb{R} \setminus \{\sqrt{2}\}$ with mathematical operations the same as for $\mathbb{R}$. Is this number system a *complete ordered field*? Explain why or why not. You do not need to write a proof, but you should show some careful reasoning.

2. (15 pts) Prove that $a_n = \frac{2n+1}{n+\sqrt{n}}$ converges.

3. (15 pts) Prove that $a_n = \frac{2n^3+1}{n+1}$ diverges to infinity.

4. (15 pts) Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence which converges to $A \in \mathbb{R}$ and $\{b_n\}_{n\in\mathbb{N}}$ a sequence which converges to $B \in \mathbb{R}$. Prove that $a_n + k \cdot b_n \to A + k \cdot B$ as $n \to \infty$ where $k \in \mathbb{R}$.