MATH 413 – Real Analysis I – Fall 2020 – HW 07

Please answer the questions below. I prefer to have your file scanned and saved as a single pdf and submitted to Blackboard.

Name the pdf file: `hw07_math413_lastname.pdf` with "lastname" of course replaced by your last name.

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1. Section 4.4 Exercise 1(d). *(Hint: First prove that $\sqrt[3]{x} - \sqrt[3]{t} \leq \sqrt[3]{x-t}$ for $x > t \geq 0$. Then prove that $f(x) = \sqrt[3]{x}$ is uniformly continuous on $[0, \infty)$ and $(-\infty, 0]$. Then since it is continuous at 0 you get that it is uniformly continuous on $(-\infty, 0] \cup [0, \infty) = \mathbb{R}$. There are other ways to solve this problem too, this is just a suggested route.)*

2. Show that $f(x) = x^2$ is uniformly continuous on $[0, 5)$. Directly us the definition of uniform continuity.

3. Show that $f(x) = \frac{1}{x-1}$ is not uniformly continuous on $(1, \infty)$. *(Hint: Find sequences x_n and t_n that work with Remark 4.4.4.)*

4. (optional, bonus) Show that $f(x) = e^x$ is not uniformly continuous on $\mathbb{R}$. *(Hint: Consult my supplemental notes on the natural exponential. Argue that $e^n > 2^n \geq n^2$ for $n \in \mathbb{N}$. Then use $t_n = n$ and $x_n = n + \frac{1}{n}$ and show that $f(x_n) - f(t_n) = e^n(e^{\frac{1}{n}} - 1)$. Then consult my supplemental notes on the natural exponential Theorem 6 that shows $(1 + \frac{1}{n})^n$ is increasing and argue that $e^{\frac{1}{n}} \geq 1 + \frac{1}{n}$. Finally, put this all together to get that f is not uniformly continuous using this sequential characterization.)*