Math 421 – Fall 2021 – Chapter 1: Combinatorics

Name:_____

Instructions: Use this for note taking and practice.

Note: Generally 'distinct' items means you can visually distinguish between them, e.g. two balls that are painted different colors or marked with different symbols. 'Indistinguishable' items or arrangements means you can not visually distinguish between them, e.g. two balls painted the same color or swapping non-distinct items.

1. Write down the *permutation* formula and give an example. How many distinct ways can we select k out of n distinguishable items and arrange them in order?

2. Write down the *combination* formula and give an example. How many distinct ways can we select a subset of size k out of n distinguishable items?

3. Write down the *multinomial* or *distinguishable permutation* formula and give an example. How many distinct ways can n distinguishable items be separated into k groups with n_i in the ith group for $i = 1, 2, \ldots, k$?

4. Write down the *multiset* formula (for the stars-and-bars problem). How many distinct ways can k indistinguishable balls be distributed among n distinguishable bins?

5. Fill in the following table with the correct formula for when sampling k items out of n total distinct options:

	without replacement	with replacement
order matters		
order doesn't matter		

6. You have 5 indistinguishable balls (imagine all are painted the same color and with no other markings) and are to distribute them among 2 labeled bins. How many distinct possibilities are there? Try drawing.

7. How many distinguishable ways are there to rearrange the letters of the word: ARRANGE?

8. There are 10 people in a room and you are to divide them into a group of 7 and a group of 3. How many distinct ways can this be done?

9. A lottery consists of randomly drawing 3 balls from a box that contains 50 balls (labeled with numbers 1 to 50). Consider the following rulesets, and write down the number of distinct possible draws there are.

   (a) The balls are selected without replacement, and the specific order they are drawn in matters.

   (b) The balls are drawn with replacement, and the specific order matters.

   (c) The balls are drawn without replacement, and the order they are drawn in doesn't matter.

10. You have 15 people in a room and 4 different books (different titles). Of the 4 book titles, you have 2, copies of each book except for the last one of which you have 9 copies. How many ways can you give out the books?

11. Consider a standard deck of 52 cards with 4 suits ($\heartsuit, \diamondsuit, \spadesuit, \clubsuit$) and 13 distinct ranks (2-10, J, Q, K, A). A full house in 5 card draw consists of 2 distinct ranks with a triple (3 suits of one rank) and a pair (2 suits of the other rank). How many distinct full houses are possible? Try doing this for 'three of a kind' and 'two pair' (these are a bit tougher).

12. If we roll five standard 6-sided dice (the dice are indistinguishable, white cubes each with sides marked with numbers 1 to 6), how many ways can we get 'three of a kind'? How about 'two pair'? How many ways call all 5 dice show different values?