1. True or False. If true, give the location where a similar statement appears in the text. If false, modify the statement so that it is true.

(a) The solution set of the linear system whose augmented matrix is \[
\begin{bmatrix}
\bar{a}_1 & \bar{a}_2 & \bar{a}_3 & \bar{b}
\end{bmatrix}
\] is the same as the solution set of the equation \(x_1\bar{a}_1 + x_2\bar{a}_2 + x_3\bar{a}_3 = \bar{b}\).

(b) The weights \(c_1, \ldots, c_n\) in a linear combination \(c_1\bar{v}_1 + \cdots + c_n\bar{v}_n\) cannot all be zero.

(c) A vector \(\bar{b}\) is a linear combination of the columns of a matrix \(A\) if and only if the equation \(A\bar{x} = \bar{b}\) has at least one solution.

(d) If \(A\) is an \(m \times n\) matrix and the equation \(A\bar{x} = \bar{b}\) is inconsistent for some \(\bar{b} \in \mathbb{R}^m\), then \(A\) cannot have a pivot position in every row.

(e) If the equation \(A\bar{x} = \bar{b}\) is inconsistent, then \(\bar{b}\) is not in the set spanned by the columns of \(A\).

2. Determine if \[
\begin{bmatrix}
2 \\
-1 \\
6
\end{bmatrix}
\] is a linear combination of \[
\begin{bmatrix}
1 \\
-2 \\
0
\end{bmatrix}, \begin{bmatrix}
0 \\
1 \\
2
\end{bmatrix}, \begin{bmatrix}
5 \\
-6 \\
8
\end{bmatrix}.
\]

3. Determine if \[
\begin{bmatrix}
3 \\
-7 \\
-3
\end{bmatrix}
\] is a linear combination of the columns of the matrix \[
\begin{bmatrix}
1 & -2 & -6 \\
0 & 3 & 7 \\
1 & -2 & 5
\end{bmatrix}.
\]

4. Let \(A = \begin{bmatrix}
1 & 0 & -4 \\
0 & 3 & -2 \\
-2 & 6 & 3
\end{bmatrix}\) and \(\bar{b} = \begin{bmatrix}
4 \\
1 \\
-4
\end{bmatrix}\). Denote the columns of \(A\) by \(\bar{a}_1, \bar{a}_2, \bar{a}_3\) and let \(W = \text{Span}\{\bar{a}_1, \bar{a}_2, \bar{a}_3\}\).

(a) Is \(\bar{b}\) in \(\{\bar{a}_1, \bar{a}_2, \bar{a}_3\}\)? How many vectors are in \(\{\bar{a}_1, \bar{a}_2, \bar{a}_3\}\)? Explain.

(b) Is \(\bar{b}\) in \(W\)? How many vectors are in \(W\)? Explain.

(c) Show that \(\bar{a}_1\) is in \(W\).

5. Similar to the example done in class, prove that \((\bar{u} + \bar{v}) + \bar{w} = \bar{u} + (\bar{v} + \bar{w})\) for all vectors \(\bar{u}, \bar{v}, \bar{w} \in \mathbb{R}^n\).

6. Let \(\bar{u} = \begin{bmatrix}
8 \\
-4
\end{bmatrix}\) and \(\bar{v} = \begin{bmatrix}
0 \\
7
\end{bmatrix}\). Compute \(\bar{u} + \bar{v}\), and \(\bar{u} - 2\bar{v}\) and draw a picture in the \(xy\)-plane of the vectors \(\bar{u}, \bar{v}, \bar{u} + \bar{v}, -2\bar{v}, \bar{u} - 2\bar{v}\).

7. For which values of \(t\) is the vector \[
\begin{bmatrix}
t \\
-5
\end{bmatrix}
\] in the plane generated by \[
\begin{bmatrix}
1 \\
0
\end{bmatrix}, \begin{bmatrix}
-3 \\
1
\end{bmatrix}\]?

8. Show that \[
\begin{bmatrix}
a \\
b
\end{bmatrix}
\] is in \(\text{Span}\{\begin{bmatrix}
2 \\
-1
\end{bmatrix}, \begin{bmatrix}
2 \\
1
\end{bmatrix}\}\).

9. Construct a \(3 \times 3\) matrix with nonzero entries and a vector in \(\mathbb{R}^3\) such that your vector is not in the set spanned by the columns of your matrix.

10. Compute the product of the matrices if possible, or explain why the product is undefined.

(a) \[
\begin{bmatrix}
-4 & 2 \\
1 & 6 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
3 \\
-2 \\
7
\end{bmatrix}.
\]

(b) \[
\begin{bmatrix}
6 & 5 \\
-4 & -3 \\
7 & 6
\end{bmatrix} \begin{bmatrix}
2 \\
3
\end{bmatrix}.
\]

11. Write the system consisting of the linear equations \(8x_1 - x_2 = 4, 5x_1 + 4x_2 = 1,\) and \(x_1 - 3x_2 = 2\) first as a vector equation and then as a matrix equation.
12. Let \( A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} \) and \( \vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \). Show that the equation \( A \vec{x} = \vec{b} \) does not have a solution for all possible \( \vec{b} \) and describe the set of all \( \vec{b} \) such that the matrix equation \( A \vec{x} = \vec{b} \) does have a solution.

13. Do the vectors \( \vec{v} = \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix} \), \( \vec{w} = \begin{bmatrix} 0 \\ 8 \\ -5 \end{bmatrix} \), and \( \vec{b} = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \) span \( \mathbb{R}^3 \)? Explain.

14. Let \( \vec{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix} \), \( \vec{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} \), and \( \vec{w} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} \). It can be shown that \( 3\vec{u} - 5\vec{v} - \vec{w} = \vec{0} \). Use this fact (and no row operations) to find \( x_1 \) and \( x_2 \) that satisfy the equation
\[
\begin{bmatrix}
7 & 3 \\
2 & 1 \\
5 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}.
\]

15. Construct two \( 3 \times 3 \) matrices, neither of which are in echelon form, one whose columns span \( \mathbb{R}^3 \) and one whose columns do not span \( \mathbb{R}^3 \). Show that the matrices you construct have the desired properties.

16. Could a set of two vectors in \( \mathbb{R}^3 \) span all of \( \mathbb{R}^3 \)? Explain. What about \( n \) vectors in \( \mathbb{R}^m \) where \( n \) is less than \( m \)?

17. Write the augmented matrix corresponding to the matrix equation
\[
\begin{bmatrix}
1 & 2 & 1 \\
-3 & -1 & 2 \\
0 & 5 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.
\]
Then solve the system and write the solution as a vector.

18. Is \( \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix} \) in the subset of \( \mathbb{R}^3 \) spanned by the columns of \( \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix} \)? Explain.