1. True or False. Justify each answer, preferably with an explanation or counterexample (as appropriate).
   
   (a) The cofactor expansion of $\det A$ down any column is the same as the cofactor expansion across any row.
   
   (b) The determinant of a triangular matrix the sum of the entries on the main diagonal.
   
   (c) The determinant of $A$ is the product of the pivots in any echelon form $U$ of $A$, multiplied by $(-1)^r$ where $r$ is the number of row interchanges made during row reduction from $A$ to $U$.
   
   (d) $\det(A + B) = \det(A) + \det(B)$.
   
   (e) If $\det A$ is zero, then two rows or two columns are the same, or a row or a column is zero.

2. Compute the determinant of
   $$\begin{pmatrix}
   1 & 2 & 4 \\
   3 & 1 & 1 \\
   2 & 4 & 2 
   \end{pmatrix}$$
by cofactor expansion across the first row and also by cofactor expansion down the second column.

3. Compute the determinant of
   $$\begin{pmatrix}
   4 & 0 & 0 & 5 \\
   1 & 7 & 2 & -5 \\
   3 & 0 & 0 & 0 \\
   8 & 3 & 1 & 7 
   \end{pmatrix}$$
using cofactor expansion (in the easiest way you can find).

4. If you haven’t seen the trick for for computing a $3 \times 3$ matrix, using a second copy of the first two columns and the diagonal products, look it up either in the exercises section of the textbook, or online. Use this trick to compute the determinant of
   $$\begin{pmatrix}
   1 & 3 & 4 \\
   2 & 3 & 1 \\
   3 & 3 & 2 
   \end{pmatrix}.$$  
**Warning:** This trick does NOT generalize in any reasonable way to larger matrices!

5. Choose a random $5 \times 5$ matrix and compute its determinant using software of your choice.

6. What property of determinants does
   $$\begin{vmatrix}
   2 & 6 & -8 \\
   2 & 0 & -3 \\
   3 & -5 & 2 
   \end{vmatrix} = 2 \begin{vmatrix}
   1 & 3 & -4 \\
   2 & 0 & -3 \\
   3 & -5 & 2 
   \end{vmatrix}$$
illustrate?

7. Find the determinant by row reduction to echelon form:
   $$\begin{vmatrix}
   1 & 3 & 0 & 2 \\
   -2 & -5 & 7 & 4 \\
   3 & 5 & 2 & 1 \\
   1 & -1 & 2 & -3 
   \end{vmatrix}.$$  

8. Find the determinant by a combination of row reduction and cofactor expansion:
   $$\begin{vmatrix}
   2 & 5 & 4 & 1 \\
   4 & 7 & 6 & 2 \\
   6 & -2 & -4 & 0 \\
   -6 & 7 & 7 & 0 
   \end{vmatrix}.$$  

9. Using the determinant, determine if the columns of the the matrix in problem 2 are linearly independent. What about set of columns
   $$\begin{Bmatrix}
   \begin{pmatrix}
   1 \\
   -2 \\
   3 \\
   1 
   \end{pmatrix}, \\
   \begin{pmatrix}
   3 \\
   -5 \\
   5 \\
   -1 
   \end{pmatrix}, \\
   \begin{pmatrix}
   0 \\
   2 \\
   2 \\
   2 
   \end{pmatrix}, \\
   \begin{pmatrix}
   2 \\
   4 \\
   1 \\
   -3 
   \end{pmatrix}
   \end{Bmatrix}?$$

10. Show that if $A$ is invertible, then $\det(A^{-1}) = \frac{1}{\det(A)}$. 

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11. Let $A$ and $B$ be square matrices. Recall that $AB = BA$ may not be true. Show, however, that $\det(AB) = \det(BA)$ is always true.

12. Let $U$ be a square matrix such that $UU^T = 1$. Show that $\det(U)$ must be either a positive or negative one.

13. Let $A$ and $B$ be $4 \times 4$ matrices with $\det A = -3$ and $\det B = -1$. Compute the following using properties of determinants:

   (a) $\det(AB)$
   (b) $\det(B^5)$
   (c) $\det(2A)$
   (d) $\det(A^T BA)$
   (e) $\det(B^{-1}AB)$