Cramer’s Rule

1. Use Cramer’s rule to compute the solution of the system:
\[-5x_1 + 3x_2 = 9\]
\[3x_1 - x_2 = -5\]

2. Find the area of the parallelogram whose vertices are \((0, 0), (-1, 3), (4, -5),\) and \((3, -2)\).

Change of Basis

3. Let \(D = \{\vec{d}_1, \vec{d}_2, \vec{d}_3\}\) and \(F = \{\vec{f}_1, \vec{f}_2, \vec{f}_3\}\) be bases for a vector space \(V\), and suppose \(\vec{f}_1 = 2\vec{d}_1 - \vec{d}_2 + \vec{d}_3, \vec{f}_2 = 3\vec{d}_2 + \vec{d}_3,\) and \(\vec{f}_3 = -3\vec{d}_1 + 2\vec{d}_3.\)
   
   (a) Find the change-of-coordinates matrix from \(F\) to \(D\).
   
   (b) Find \([x]_D\) for \(\vec{x} = \vec{f}_1 - 2\vec{f}_2 + 2\vec{f}_3.\)

4. Let \(B = \left\{ \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix} \right\}\) and \(C = \left\{ \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}\) be bases for \(\mathbb{R}^2.\)
   
   (a) Find the change-of-coordinates matrix from \(B\) to \(C\).
   
   (b) Find the change-of-coordinates matrix from \(C\) to \(B\).

5. In \(\mathbb{P}_2\), find a change-of-coordinates matrix from the basis \(B = \{1 - 2t + t^2, 3 - 5t + 4t^2, 2t + 3t^2\}\) to the standard basis \(C = \{1, t, t^2\}\). Then find the \(B\)-coordinate vector for \(-1 + 2t\).

Complex Eigenvalues

6. Let the matrix \(A = \begin{bmatrix} 4 & -5 \\ 1 & 2 \end{bmatrix}\) act on \(\mathbb{C}^2.\) Find the eigenvalues and a basis for each eigenspace in \(\mathbb{C}^2.\)

7. Let \(A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\) which has eigenvalues \(\lambda_1 = 1 + 2i\) and \(\lambda_2 = 1 - 2i\) and corresponding bases for the eigenspaces the single vectors \(\vec{v}_1 = \begin{bmatrix} -2i \\ 1 \end{bmatrix}\) and \(\vec{v}_2 = \begin{bmatrix} 2i \\ 1 \end{bmatrix}.\)
   
   (a) Find the similar matrix \(C.\)
   
   (b) Using the similar matrix \(C\) find the scale factor \(r.\)